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1826

1956

Some Prominent Members of The Franklin Institute
3. Matthias William Baldwin, 1795-1866
THOMAS COULSON

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CYRIL P. ATKINSON AND L. O. HEFLINGER

Reliable Circuits Using Less Reliable Relays
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R. E. ROBERSON AND D. TATISTCHEFF

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VOL. 262 NO. 3

FOR

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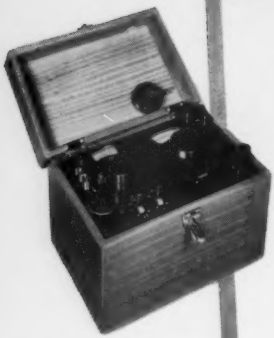

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INDEX TO ADVERTISERS

	PAGE
Ajax Electrothermic Corp.	ii
Almo Radio Co.	v
Atlas Photo Engraving Co.	x
Bell Telephone Laboratories	vi
Biddle Co., James G.	viii
Bolger-Parker Co.	x
Buten, M. & Sons	v
Coleman, W. B., & Co.	ix
Damon & Foster	ix
Davidson, P. L.	ix
Eggly-Furlow Engineers	ix
Fehr & Johnson, Inc.	x
Fidelity-Philadelphia Trust Co.	xii
Franklin Institute, Awards	xi
Franklin Institute, Laboratories	Fourth cover page
Franklin Institute, Membership	Third cover page
Harris-Dechant Associates	ix
Herbach & Rademan	x
Hess & Barker	viii
Kearney Lumber Co.	x
Lancaster Press, Inc.	iv
Leeds & Northrup Co.	Second cover page
Leopold, Charles S.	ix
Levy & Co., Max	v
Shanahan & Co.	x
Steinberg and Company, Albert	v
Thomas Co., Arthur H.	viii
Yarnall-Waring Co.	iv

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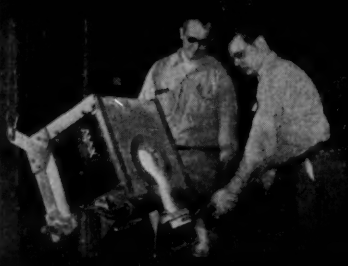

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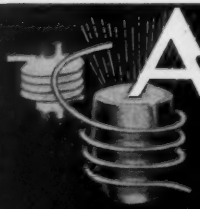

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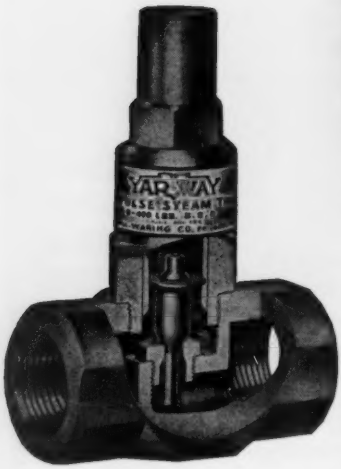

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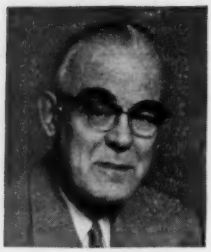

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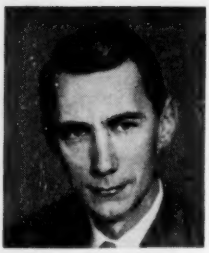

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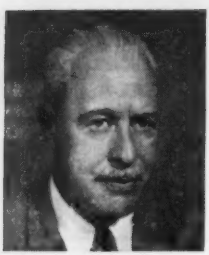

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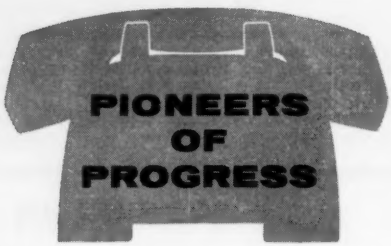

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Vol. 262 **SEPTEMBER, 1956** **No. 3**

CONTENTS

Some Prominent Members of The Franklin Institute. 3. Matthias William Baldwin, 1795–1866 ..... THOMAS COULSON 171

Subharmonic and Superharmonic Oscillations of a Bilinear Vibrating System ..... CYRIL P. ATKINSON AND L. O. HEFLINGER 185

Reliable Circuits Using Less Reliable Relays, Part I ..... E. F. MOORE AND C. E. SHANNON 191

The Potential Energy of a Small Rigid Body in the Gravitational Field of an Oblate Spheroid ..... R. E. ROBERSON AND D. TATISTCHEFF 209

Notes from the National Bureau of Standards 215

The Franklin Institute
- Library 220
- Journal of The Franklin Institute 222
- Publications Received 222
- Notes from the Franklin Institute Museum 223
- Notes from the Biochemical Research Foundation 225
- Notes from the Bartol Research Foundation 229

Book Reviews 230

Book Notes 234

Current Topics 238

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The Newcomen Medal (1943—Gold Medal).—This medal is awarded, not oftener than once in three years, for achievement in the field of Steam.

The Francis J. Clamer Medal (1943—Silver Medal).—This medal is awarded at least once in five years for meritorious achievement in the field of Metallurgy.

The Stuart Ballantine Medal (1946—Gold Medal).—This medal is to be awarded in recognition of outstanding achievement in the fields of Communication and Reconnaissance which employ electromagnetic radiation.

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Vol. 262 SEPTEMBER, 1956 No. 3

SOME PROMINENT MEMBERS OF THE FRANKLIN INSTITUTE
3. MATTHIAS WILLIAM BALDWIN, 1795–1866

BY

THOMAS COULSON[1]

Matthias William Baldwin, manufacturer and philanthropist, was born in Elizabethtown, N. J., on December 10, 1795. William Baldwin, his father, was a prosperous carriage-maker. He had displayed more than average ability in pursuing his trade, and his personal qualities had gained for him a wide local respect. He died when Matthias was only four years old. Although he left a considerable property, the imprudent management of those who administered his estate dissipated the greater part, leaving the widow and young family to enjoy the doubtful blessings of honorable poverty.

Napoleon's comment that "the destiny of the child is always the work of the mother" was fully borne out in this case. Mrs. Baldwin proved to be a courageous and sensible woman who guided the upbringing of her children under trying conditions. She guided her children, of whom Matthias was the youngest of five, in the paths of thrift, ingenuity, and independence.

The schools of the day made no pretense of making learning attractive to young people. Matthias, having acquired the rudimentary education offered by the local school, showed no particular desire to supplement the bare-bones of fact offered him in the classroom. From his earliest years it was evident the bent of his mind was essentially practical. The time a studious boy would normally have spent with his books, or a slothful one at play, was devoted to any piece of mechanism that chance threw in his way. It was told of him as a child that he made a practice of taking all new toys apart, a trait not uncommon in childhood, but where he proved to be exceptional was his ability to reconstruct them, often with boyish improvements.

[1] Director of Museum Research, The Franklin Institute, Philadelphia, Pa.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.

Little else has survived from his earliest days that would assist in completing the picture of his education. His mother did what she could to mold the boy's character while keeping his spirits active and free. He was brought up with a stern respect for the value of other people's property. In manhood, when his integrity was praised, he recalled how once, having need of a brick for one of his childish projects, he had borrowed one from a pile deposited nearby by a builder. His mother made him return it, pointing out that he was not really borrowing but stealing the brick. Thus, he was well grounded in that scrupulous morality that was to be so conspicuous in his subsequent business dealings.

The mother showed a remarkable insight into the boy's character and displayed appreciation for his latent talents by encouraging him to whittle toys, and by permitting him to scatter whittlings and filings liberally about the house. It is to be remarked of him at this stage, he was never content with shoddy workmanship. Whatever he made had to be precise and finished. The works of clocks, until recently made of wood, and watches fascinated him, as they did many other boys at a time when mechanisms were rare. Naturally, he hoped to find some employment where his passion for accurate workmanship could find expression, but as there were few occupations in those days demanding this qualification, he was fortunate in becoming apprenticed to Woolworth Brothers, a jewelry firm in Frankford. This implied his departure from home, and caused him to seek his home with the family of a married fellow-worker, where he remained until his apprenticeship was completed in 1817. By this time his mother had taken up residence in Philadelphia.

In this same year he became a journeyman in the employ of Fletcher and Gardener, whose manufacturing jewelry business was conducted at 130 Chestnut Street, Philadelphia. Here he quickly established a reputation for the excellence of his craftsmanship. Among his fellow-workers and associates he was remarkable for his reticence and obstinacy. He was of an independent disposition, too, for after two years with these employers he decided upon entering business on his own account. He did not prosper. Later in years he declared that he "could not spend his life making 'gew gaws'" but the approach to the financial depression of 1825 had an introductory period when the making and repairing of jewelry was unprofitable, and this may have influenced his decision to try another trade.

There can be little reason to doubt that his training in the fine work of a jeweler influenced all that he did in later life. Apart from the manual dexterity it demanded the work gave expression to his love for beautiful things. But principally it was reflected in the insistence upon meticulous performance in any undertaking, and the employment of the best tools obtainable. A friend said of him "He was

more than artisan—always artist" (1).[2] Thus far, the only sign of being an innovator was given in a vague allusion to the development of a method of gold-plating that later was commonly used by his friends in the trade. What the method was cannot now be identified.

However, it is evident that Baldwin was thinking deeply at this time of working on baser materials than gold and gems. Among his closer acquaintances was David Mason (brother of William Mason, later the drawing instructor at The Franklin Institute) who described himself as a mechanic but who was a wood engraver. In the course of a conversation Mason lamented that all the tools used in his trade were of foreign make. This led to the agreement that the tools could be made equally well in this country, and further conversation and discussion conducted them eventually to the decision they should start a business in which they would undertake to make all the tools used by wood engravers and book binders. In the year 1824 they rented premises on Bank Coffee House Alley, off Walnut Street, and between Fourth and Fifth Streets.

The enterprising Baldwin soon perceived a direction in which the business could be expanded. To the hand tools they had been making he added the hydraulic presses used in book binding, and the rolls used for calendering paper. Between them, the partners devised a new method for printing calico. The general practice was to stamp the patterns from plane blocks. Baldwin and Mason built a cylinder machine with the design engraved on the copper roller. The bolt of calico passed between the rollers in a continuous motion, thereby greatly accelerating the process of printing. Shortly afterwards Baldwin introduced an improvement for transferring the design to the surface of the printing cylinder or roller. Formerly, this task had involved so much handwork that Baldwin realized it could be greatly simplified by conversion to a mechanical process. The practice he sought to replace comprised building up the design by punches. He decided upon engraving the design on soft steel which was then hardened, and the matrix was employed to transfer the design to other rollers made of soft metal (2). At a still later date the process was further improved to print in three colors.

Realizing that he was in need of acquaintance with the science of mechanics, Baldwin was an eager collaborator in the efforts of Samuel Vaughan Merrick and Professor Keating in the formation of a society which would promote education in the subject. He attended the meeting in Carpenters' Hall which resulted in the founding of The Franklin Institute. He signed the first roll of members, describing his occupation as "machinist." He maintained his interest in the Institute for the remainder of his active life, serving on the Board of Managers from 1827 to 1863.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

With the manufacture of these printing rollers supplementing their other products, the partners became prosperous, busy, and crowded in their small quarters. They were compelled to seek a larger establishment which would accommodate their growing numbers of workmen and provide space for handling the larger materials they were making up. The firm moved into more spacious quarters on Minor Street. Here, they were faced with the problem of a power supply. In their former premises they had experimented with one horse-power—and that on four legs—but this was inadequate for current needs. A substitute was provided in the purchase of a small steam engine.

In those days steam engines were heavy, cumbersome, and noisy. Baldwin with his passion for neatness and efficiency found this one a bugbear. Stimulated by the protests of neighbors, who loudly declaimed against its noise and vibration, he tinkered with its mechanism in the expectation that he might improve its efficiency and reduce its noise. His efforts were so unavailing that he abandoned hope of improving the old engine and decided upon building one of his own design that would eliminate the defects he had tried to remedy.

The steam engine he designed and built was remarkable in more than one respect, but it principally found favor (especially in the neighborhood of Minor Street) because it was almost noiseless in operation. While this feature might have appealed to the other tenants of the building, engineers will be interested in its other features, most noteworthy being the vertical cylinder and the yoke type of connecting-rod invented by Baldwin for reducing the over-all height. Although the engine developed six horse-power it is unusually compact as compared with its contemporaries. It must have been sturdily constructed, too, for it was later transferred to the boiler shop of the Baldwin Locomotive Works, where it continued in operation until 1873. It is now exhibited in the Smithsonian Institution.

Baldwin's first stationary engine was so greatly admired that he received orders to make similar engines in sufficient quantity to justify him in believing he could add an engineering section to the business. This perception of the trend of the times toward the substitution of steam for water power was too radical for Mason. He was frightened out of the partnership at the prospect of embarking upon such alien interests. It was probably as well David Mason withdrew when he did, for his conservative mind would have recoiled in horror from the next move of his partner.

By this time the United States had passed through two phases of the struggle to improve her internal communications. The nation had been feverishly building roads and turnpikes until the Erie Canal diverted attention into the cheaper channel of water transportation. Great activity was being displayed in expanding these two important means of knitting together distant communities and for providing an

expanding interior with an outlet to the sea. Two such innovations were regarded as a desire to furnish the latest patterns of transportation, when suddenly came news of the successful steam railroads in England.

Railways were not uncommon near coal mines. The rails were usually of hickory wood, sometimes plated with iron. The locomotive power was supplied by horses. The problems of inventors were concentrated upon the track long before it was believed possible steam engines would run upon it. As the nineteenth century opened, a number of experiments were being made with steam carriages. Oliver Evans produced a monstrosity which had been designed as a river dredge. A few other men were actively advocating the use of steam, without gaining much support.

Was it the eager ghost of Franklin that beckoned them to Philadelphia? John Fitch, Oliver Evans, John Stevens, among others, sought the approval of his American Philosophical Society for their ideas. Stevens and Benjamin Dearborn founded the ponderously named Pennsylvania Society for the Promotion of Internal Improvements in the Commonwealth, which explained the railroad to America. In 1824 this group sent its engineer, William Strickland, to England to bring back all the information he could gather. He found a great deal, and he presented it to the public in a magnificent volume (3).

The publication of this report created a furor. Out of the hurly burly of the press comments came much fantasy and not much sane comment, until arrival of the news that the Rainhill trials had demonstrated the practicability of the steam railroad. The newest form of transportation fired the imagination of the American people. Demand for it became the popular topic of the day. One enterprising firm of distillers showed the thought of the day by picturing a steam locomotive on its whisky bottle labels.

No one in Philadelphia had yet seen a steam locomotive. Franklin Peale was proprietor of a museum in the Arcade on Chestnut Street and, while he may not have been the original American showman, he was a pioneer in the best tradition of the flamboyant profession. He made a feature of having the latest sensation on display, whether it were the bones of a recently disinterred dinosaurus or that latest freak in home lighting with coal gas. Now that everyone was talking about steam railroads he had to have one to attract visitors to his museum.

His friend Baldwin was now recognized as an authority on the steam engine. Peale, therefore, appealed to him for a model locomotive and passenger cars. With nothing to guide him but the imperfect descriptions of the English locomotives, Baldwin consented and began to build. However, the available material had too many unexplained details to permit a rapid construction of the model. Experiments and modifications were imperative before the model could be completed, but on

April 25, 1831, it was put into operation at the museum to the delight of the visitors. The locomotive ran on a circular track and hauled two cars with seats for four passengers.

The success achieved by this model encouraged the managers of the Philadelphia, Germantown, and Norristown Railroad to approach Baldwin with the request that he build them a full-scale locomotive. This railroad had not been operated by steam, its cars having been hauled over the rails by horses, but the managers being forward-looking men were jealous of the progress being made across the Delaware River in New Jersey. We have no means of knowing whether Baldwin gave a ready affirmative to the proposal or whether he withheld his consent until he had a better conception of what was involved. He had not yet seen a practical locomotive, but an opportunity for seeing one was at hand.

The Camden and Perth Amboy R.R. had just imported a locomotive from England that had been smuggled into a shed at Bordentown, N. J., where it was placed under guard. By means known only to themselves, Baldwin and Peale overcame the scruples of the guardians and were permitted to examine those parts of the locomotive that had not been assembled. Baldwin made a few measurements and came away confident he could build a locomotive.

Apart from his inexperience in constructing any comparable piece of mechanism, Baldwin had to overcome some physical handicaps that might have discouraged a man of less resolute character. He lacked the tools and had no place to do the heavy forging required. The larger parts had to be fabricated in other plants. Meanwhile he had to find larger premises where these parts could be finished and assembled. These he found in Lodge Alley, between Seventh and Eighth Streets, backing onto Market Street. Here, despite all drawbacks the locomotive, "Old Ironsides," was completed in about six months. The construction was partly of wood and partly of iron. It weighed six tons and was capable of drawing a load of 30 tons at a speed of 28 miles an hour over level ground. The trial run was held November 23, 1832. This apparently was a triumph, loudly acclaimed by the spectators. One of the newspapers described the event in the following words (4)

> It gives us pleasure to state that the locomotive engine built by our townsman, M. W. Baldwin, has been proved highly successful. In the presence of several gentlemen of science and information on such subjects, the engine was yesterday placed upon the road for the first time. All her parts had been previously highly finished and fitted together in Mr. Baldwin's factory. . . . The placing fire in the furnace and raising steam occupied 20 minutes. The engine (with her tender) moved from the depot in beautiful style, working with great ease and uniformity. She proceeded about half a mile beyond the Union Tavern, at the township line, and returned immediately, a distance of six miles, at a speed of about 28 miles to the hour; her speed having been slackened at all the road crossings, and it being after dark, but a portion of her power was used. It is needless to say the spectators were delighted.

However, there is reason to doubt whether the new owners regarded their locomotive as more than an advertising allurement to tempt travellers into having their first journey on a steam railroad. They placed most reliance upon their horses. It is true that under the control of its designer and maker, the trial run had been successful, but that the owners had some scepticism may be gathered from their announcement (5) in the newspapers.

> NOTICE:—The locomotive engine built by M. W. Baldwin of this city will depart daily, when the weather is fair, with a train of passengar cars. On rainy days horses will be attached.

Weather was not the only obstacle that had to be overcome. Trouble developed on the first public run, possibly because no experienced engineer could be found but, possibly, because of a mechanical defect. The latter reason is implied, as Baldwin himself was occupied for some days in getting the locomotive back into operation. It seems hardly likely that an unqualified engineer could have damaged it to that extent. More than likely, the foundation of the trouble resided in the light weight of the locomotive. The fact that it was so light is assigned to the fault of the railroad officials who had specified its weight. It should be recalled that the rails were wooden and unsuitable for the task of carrying a steam locomotive. In any event, the officials were dissatisfied with the performance, and declined to pay the price arranged.

The cost of the locomotive had been agreed upon as $4000, but the company claimed that it did not perform in accordance with the contract conditions. Eventually, a compromise was reached, and Baldwin was paid $3500. This discouraging proceeding provoked him into declaring it was the last locomotive he would ever built. He repented, later, and at the time of his death, the works he formed had its 1500th locomotive under construction.

"Old Ironsides" came to an untimely end in a landslide that destroyed her.

Still, he allowed two years to pass before he was tempted to try again. In spite of the disappointment caused by his first locomotive he had not lost interest in the steam engine, and he continued to cultivate friendships with other engineers. One of his acquaintances was E. L. Miller, who had designed the "Best Friend," a locomotive which abruptly passed into railroad history when a jittery negro fireman, exasperated by the noise of escaping steam, tied down the safety valve. Acting upon Miller's advice, Baldwin went to inspect one of Stephenson's locomotives just imported by the Mohawk and Hudson R.R. (6). As a result of this inspection he became convinced he could build a superior design. He worked very quickly, for he obtained a patent on the "half crank" September 10, 1834, which was a marked improvement.

This model appealed to Miller, who ordered one for the Charleston and Hamburg R.R., which was completed in February, 1834. In June he had ready another for the State Road of Pennsylvania. The performance of this latter locomotive was unprecedented. It immediately established Baldwin's reputation as a locomotive builder. Before the year was out he was building five more of this type. His pessimism over the difficulties created by "Old Ironsides" vanished on the wave of this prosperity. He was now definitely committed to the business of locomotive building.

Benefitting from his experience with the first, he was building all his locomotives on a larger and heavier scale. This had caused more congestion in the Minor Street building. Another removal was imperative, for he was planning upon enlarging the further locomotives he might be asked to build. A site was chosen on North Broad Street at Hamilton Street, which was thought would serve the purpose. A three storey L-shaped brick building was erected, and all activities transferred to it in 1835. In spite of his revived interest in locomotives, Baldwin continued to construct stationary engines for the next fifteen years.

In 1837, he was awarded the contract for constructing the ponderous machinery for the city's ice-breaking boat. This work was successfully accomplished and it might have been supposed to justify the maker in seeking additional contracts for steamboat machinery. Baldwin may have contemplated such a course, but he must have hesitated entering into competition with the Southwark Foundry, established by his friend Samuel Vaughan Merrick, and which had been furnished with the most formidable equipment of the very latest design. Merrick was not interested in building locomotives, a field in which Baldwin was now specializing.

Baldwin now had an energetic competitor in William Norris, but he continued to receive orders for his locomotives. Fourteen were built in 1835, forty in 1836, forty in 1837, twenty-three in 1838. The abrupt decline in the last year was an aftermath of the grave financial crisis of 1836–7, which at one time threatened to engulf Baldwin. His embarrassment grew so acute he was compelled to call a meeting of his creditors, to whom he offered to surrender all his property, business and private. As an alternative he asked that he be permitted to carry on his business with the undertaking he would pay all accounts with interest in three years. This proposal was accepted, but additional disturbances on the financial market forced an extension for another two years before he was in the clear. In this way, Baldwin managed to survive the catastrophe which overwhelmed so many of his fellow-townsmen.

Absolute and unreserved honesty was instinctive with him. This characteristic is to be observed in all his dealings, but never to better

advantage than on this occasion. When most men reasoned that an equitable compromise with their creditors was all that could be demanded in view of the general financial conditions, Baldwin paid all claims against him in full, well knowing that such a course would demand heroic sacrifices on his part. But he never compromised with principles.

While thus harrassed with financial problems Baldwin continued to seek for improvements in his locomotives. He also began to introduce manufacturing methods that permitted the parts to be interchangeable, although he was so early in this that some time had to elapse before the use of standard gauges was fully installed.

Because of his practical wisdom and his devotion to high ethical principles, he earned the trust of his fellow citizens. This imposed upon him a political duty he would gladly have avoided. His growing business responsibilities were enough to command his full attention but, at the insistence of friends, he consented to allow his name to be submitted, and he was elected to serve on the Convention that met in 1837 for the purpose of amending the State Constitution. As was to be expected from a man of his reticence he did not play an active part in the debates, rising to his feet on one occasion only and that to support a motion that the meetings be opened with prayer (**7**). He voted consistently with the minority who favored giving the vote to free colored citizens, and he did some active, if ineffective, lobbying upon this much discussed question.

His interest in promoting the well-being of the colored people was not restricted to these efforts to secure the vote for them. In 1835 he had founded a school for colored children, and paid the teachers for two years, until his circumstances caused a temporary restraint upon his philanthropic efforts.

Baldwin declined to be beguiled into running for public office. It is doubtful that he could have found time for any office to which he might have been elected, for in those days business was very much a matter of direct personal management. Baldwin is said to have personally designed every locomotive he built, and that he drew the designs of the various parts in chalk on the floor of the plant (**8**). All the processes through each stage of manufacture received his personal supervision.

During the time of the monetary depression, he worked assiduously to design improvements for his locomotives but, while he was fruitful in minor ideas, none was of major importance. Then, the idea flashed through his mind how he might overcome the one problem that surpassed in urgency all others at that time—that of utilizing the weight of the locomotive to provide adhesion and yet have a satisfactory wheel arrangement for negotiating the short curves that were characteristic of American railroads. Tradition attributes his discovery to an in-

spiration that came to him during the course of a sleepless night. The plan, so long sought after, which was to prove more influential than any other of his inventions in contributing to the fortunes of his firm, was the familiar six-wheels connected locomotive with the four front drivers combined in a flexible truck. He secured a patent for this design in 1842 (9).

The first locomotive built to this design was completed in December, 1842. It weighed twelve tons and drew, beyond its own weight, 250 tons up a grade of 36 feet to the mile. This new locomotive found immediate favor with railroad engineers, as its loads exceeded anything known in American practice, and it heralded the means of reducing operating expenses which made it popular with the executives. Prosperity returned to the Baldwin shops after several lean years.

Faster locomotives were receiving much attention as the railroads demanded them to meet competition and to serve them under the increasing pressure of an expanding industry that required speedy transportation. This was in keeping with the reckless pioneer spirit which opened up the continent. Hardy spirits were eager about "getting there." Less inclined to wait for the safest way of getting anywhere, they pleaded for more speed with a disregard for protecting people or property.

In the year 1848, the Vermont Central R.R. challenged Baldwin to build a locomotive that would run sixty miles an hour. As the handsome sum of $10,000 was offered as an inducement, Baldwin accepted the challenge. The outcome was the famous "Governor Paine." Officials of the railroad boasted this locomotive could be started from a state of rest and run a mile in forty-three seconds (6).

These successes did not satisfy the ambitious maker. He continued to strive for further improvements. When the link motion was introduced into this country in 1849, he was opposed to it on theoretical grounds that seemed important to him when translated into practice. That it provided the advantage of cutting off steam at any point in the stroke was too significant to be overlooked. Baldwin applied his intelligence to this problem. One of the first results of his investigation was the Variable Cut-off (10), which had a limited application before he finally succumbed to the claims of Stephenson's link motion. This device, in the minds of many engineers, is the dividing line between the early and the modern steam locomotive. Once his obstinate mind yielded to its superiority, Baldwin played his part in perfecting the various operational parts.

The outbreak of the Civil War was feared as an indication of the slackening in demand for locomotives and, when orders fell off in the opening stages of hostilities, Baldwin contemplated converting the plant for the production of war materials. This proved unnecessary, however, as the railroads were to play a vastly more important part in

military strategy than had been suspected, and the government placed sufficient orders, supplemented by those of the coal companies, to keep the plant fully occupied building locomotives during the succeeding years of the conflict. During this period heavier locomotives were built, and steel was introduced into their construction to a much greater extent.

The introduction of steel tires was, at first, opposed on the grounds that, as they had to be imported, there would be a hazard in the delays caused by their procurement. Baldwin displayed foresight in ordering a larger quantity than would meet his immediate needs, and holding them in stock. The firm was thereby spared from any delay in the delivery of the locomotives for which steel tires were specified.

During the war years great advances were made by the firm in the use of interchangeable parts by the general employment of standard gauges and templates.

While he was thus engaged in these improvements that made the name of Baldwin known throughout the nation, the man who had built up the reputation as the greatest American builder of locomotives was also building monuments to his faith. He became known as the founder of churches. The natural growth of the city in which he lived created a demand for more places of worship. Baldwin was distinguished for the handsome contributions he made from his own pocket toward the satisfaction of these needs, and for the activity he displayed in canvassing his friends with persuasive pleas for their practical sympathy.

There were numerous examples of his benevolence. In addition to his numerous gifts for church extension, he expended thousands of dollars on private charities of which there is no record.

The expanding business of which he was the leader and inspiring force, absorbed so much of his time there was little left for recreation or public service. He did manage to find time to acquire a proficiency in archery, and he became an accomplished horseman. The little indulgence he allowed himself in these exercises enabled him to maintain a fine athletic appearance. He was an ardent temperance advocate. His shrewd mind found expression in concise and direct speech, as in former years, and his views were decided and positive. Doubt was as foreign to his mind as dishonesty was to his practice.

He was a man of humane instincts who took a profound interest in men less fortunate than himself. For many years he served on the Board of Inspectors of the Philadelphia County Prison. In this office his experience with his workers, his innate sense of justice, and his love of mercy made him eminently useful. In those days the Inspectors had the power to discharge prisoners held for petty offenses when such act appeared to be justified. Baldwin had a good-natured weakness for the veterans of the Civil War, and he was often persuaded to exercise this prerogative on their behalf.

Baldwin also took a great interest in the work of the Sanitary Commission, that organization which took the place of the Red Cross in attending to the physical welfare of the wounded soldiers in the Civil War. He attended the inaugural meeting of the Commission in Philadelphia, and served on the executive committee until the time of his death. His financial contributions to its work were on a liberal scale.

In the year 1854, he had taken Samuel Baird into partnership, an association that endured until the end of Baldwin's life. As this relieved him from some of his responsibilities, Baldwin was able, in 1860, to indulge himself in the only long vacation he had in all his life. He travelled over Great Britain and extensively over the European continent. He divided his time between inspecting improvements in machinery and in admiration of the art treasures of Europe.

This love of art was an enduring interest, which he extended to the use of his tools. Although he did not actively pursue the practice of either music or painting he elevated the work of his hands to an art. A small bench was installed convenient to his office and, in the intervals of business, he would seize a few minutes of relaxation in renewing his skill as a jeweler. He delighted in making jewelry and small objects of beauty that appealed to his fancy. His friends were quick to observe he could not reject an opportunity to repair a watch or a piece of damaged jewelry.

His fondness for art had caused him to collect a number of oil paintings, but there was no room in his modest home for their display. His European tour was largely planned for the purpose of acquiring more pictures because he was contemplating moving into a more commodious home. Friends had often wondered why, after his marriage to Sarah C. Baldwin (a remote relative) he had continued to reside in such humble surroundings, first at Tenth Street, and later on Spruce Street, neither of which was regarded as a suitable location for a man of his position. At length he purchased a fine old-fashioned brick home on Chestnut Street at Eleventh, which had been occupied by the Union League. Although this was also surrounded by business premises, Baldwin defended his choice of the location with the argument that he preferred to live among his fellow-men rather than in splendid isolation.

Into this home he moved his family (he had a son and two daughters) and his collection of pictures, proposing to enjoy the proximity of those beautiful things which, as a young man, he had been compelled to regard from afar. Nor was this entirely self-indulgence. He was sincere in his wish to live surrounded by fellow-creatures and he was equally desirous of sharing the enjoyment of his art treasures with them. Visitors were always welcome, and to encourage them he provided music on winter evenings. These gatherings had started with the presence of personal and business acquaintances, but the gatherings grew in extent and Baldwin was forced to admit that he did not know

one half of those who were in attendance. His recognition as a patron of the fine arts came with his election to the board of managers of the Pennsylvania Academy of Fine Arts.

The crowning feature of this stately home was the conservatory, filled with tropical plants and rare flowers. Again, wishing to share beauty with those who could not acquire it for themselves, he caused the conservatory to be built so that it was fully exposed to the view of passers-by. The gardener was instructed to keep the glass clear of all moisture vapor so that the view of the interior should not be obscured. Solicitude for the passers-by was not without its drawbacks, as the household staff were made aware by the frequent ringing of the doorbell by people who were under the mistaken impression the blooms exposed were for sale.

Finally, he yielded to the desire for a country house where he could satisfy his interest in horticulture. The selection of a site could hardly be called judicious. He chose a section of marshy wasteland called Wissinoming, a little beyond Frankford, where he had been accustomed to take his solitary walks during the days of his apprenticeship. He proposed to convert this waste into a beautiful garden where he might end his days in happy retirement. There was, however, to be no repose for him. Whatever time he could snatch from business was devoted to building a spacious home, the draining of the marshes, and the laying out of the garden. He embellished the grounds to the utmost and, having planted it to his satisfaction, he added a collection of birds and small animals. He had been chosen as president of the Horticultural Society in 1858 and continued to serve in this capacity until 1863.

His life of unusual energy and well-distributed activity ended tranquilly in 1866.

In mental character, Baldwin was a man of resolution and firmness. This trait was especially characteristic in his engineering career, where it was displayed in the persistence with which he would labor at some improvement he thought was necessary. It was also apparent in his stubborn resistance to some of the innovations introduced by other men in the field. If he was led on occasion to oppose a feature of locomotive construction, it was not because he was opposed to change. He had no prejudice against change. His attitude was molded only after careful thought had produced the conviction that the proposal was bad engineering.

Baldwin's claim to remembrance does not rest alone on the outcome of his mechanical achievements. He was a man of sterling integrity, who was bound to exercise influence upon those with whom he was brought into daily contact. He was known as a philanthropist with the most generous impulses. The contribution he made to church building by no means exhausted his benevolence. He was known to put his name to notes for large sums of money for charity at times of

financial stringency. His munificence grew as his means increased. Indeed, throughout his whole life he was zealous in the performance of good deeds. And for that he should be remembered.

REFERENCES

(1) WOLCOTT CALKINS, "Memorial," Philadelphia (privately printed), 1867.
(2) As the Patent Office did not issue specifications of patents at this date, the reader is referred to the short specification that appears in the JOURNAL OF THE FRANKLIN INSTITUTE, Vol. 5, p. 418 (1828).
(3) WILLIAM STRICKLAND, "Report on Canals, Railways, and other Subjects," Philadelphia, 1826.
(4) *Chronicle*, November 24, 1832.
(5) *Poulson's American Daily Advertiser*, November 26, 1832.
(6) "History of Baldwin Locomotive Works, 1831–1881," Philadelphia, p. 10.
(7) *Proceedings and Debates of the Convention*, Vol. VI, p. 7
(8) "Philadelphia: story of progress," New York, Vol. 2, p. 366, 1941.
(9) U. S. Patent 2759, August 25, 1842. John B. Jervis had invented the two-wheel swivel or bogie truck in 1831.
(10) U. S. Patent 10,007, September 13, 1850.

SUBHARMONIC AND SUPERHARMONIC OSCILLATIONS OF A BILINEAR VIBRATING SYSTEM

BY

CYRIL P. ATKINSON [1] AND L. O. HEFLINGER [2]

ABSTRACT

This paper presents the experimental response curves of a bilinear vibrating system as produced by an electronic analog. The existence of superharmonic and subharmonic components in the responses is shown. The relation between both superharmonics and subharmonics and the natural frequency range of the free vibrations of the system is suggested as a possible approach to an understanding of the mechanism for their (the harmonics) generation. Numerical values of the circuit parameters for a particular "run" of the analog are given in an appendix.

INTRODUCTION

The existence of both subharmonic and superharmonic responses in nonlinear vibratory systems has been demonstrated (1,2,3,4).[3] Most of these works have been concerned with systems whose dynamical equations were of the Duffing type where the nonlinearity is a cubic restoring force, with the system being disturbed by a sinusoidal driving force.[4] Discussions of results (exclusive of fundamental responses) are concerned principally with the harmonic relations between the response frequencies and the driving frequency (5) (for example, a subharmonic is a submultiple of the driving frequency). The authors of this paper feel that attention may profitably be directed to the relations between the response frequencies and the natural frequencies of the non-forced system. This approach, it is thought, will lead to an understanding of the physical mechanism responsible for the production of responses at frequencies other than the driving frequency.

The purpose of this paper is to discuss a simpler system than Duffing's, that is a bilinear system which is characterized by the restoring force shown in Fig. 1. Such a bilinear system is shown to exhibit the phenomena of subharmonic and superharmonic responses. It is proposed herein that these subharmonic and superharmonic responses can be described as natural frequencies of the system which are sustained by the forcing function when its frequency is correctly related to the "natural frequencies" of the system.

The system studied for this paper was an electronic system utilizing an inductance, resistance, and a capacitor with an auxiliary loop to produce the nonlinearity. The circuit diagram is shown in Fig. 2.

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

[4] Duffing's Equation is of the form $a\ddot{x} + b\dot{x} + cx + dx^3 = f \cos \omega t$.

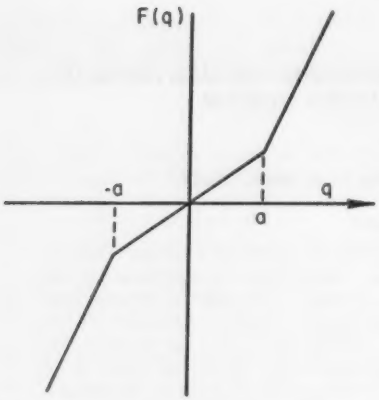

FIG. 1. Restoring force for bilinear system. $F(q)$ vs. q.

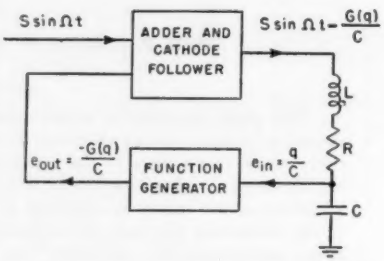

FIG. 2. Electronic analog for bilinear system.

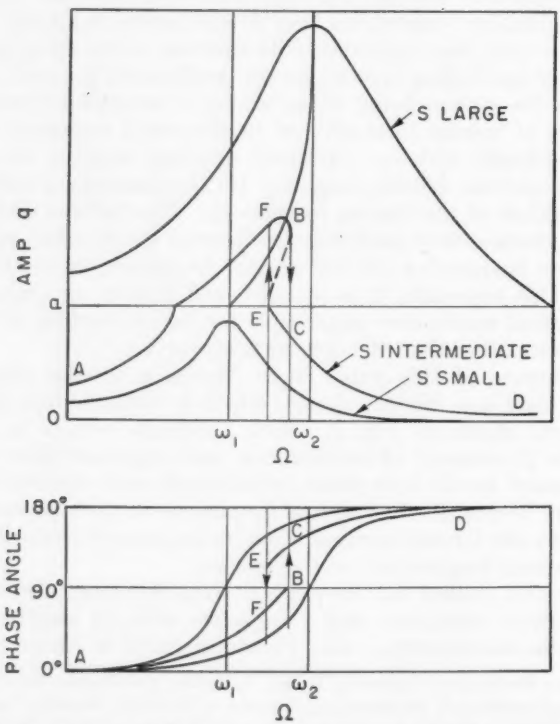

FIG. 3. Fundamental amplitude and phase response curves for bilinear system.

The function generator produces $G(q)$ the bilinearity of the system. The equation of the system is:

$$L\ddot{q} + R\dot{q} + \frac{1}{C}F(q) = S \sin \Omega t \tag{1}$$

where q is the charge on capacitor with capacitance C, R is the resistance, L is the inductance, S is the amplitude of the forcing function $S \sin \Omega t$, Ω is the circular frequency of the forcing function, t is time, and $F(q)$ is the bilinear restoring force $q + G(q)$ (see Fig. 2).

The mechanical analog of this electronic system is a damped spring mass system with a sinusoidal forcing function. The differential equation of the mechanical system has the form:

$$m\ddot{x} + b\dot{x} + k(x + G(x)) = F_0 \sin \Omega_m t. \tag{2}$$

FUNDAMENTAL RESPONSES

The electronic analog of Eq. 1 produced the response curves shown in Fig. 3. When the amplitude S of the forcing function is small, the response q is within the range $-a < q < a$ and thus the system behaves exactly as does a linear system. The natural frequency for small amplitudes of the undamped nonforced system is the same as for a linear system. It is dependent on the slope of the central segment of the $F(q)$ vs q curve (Fig. 1) and in particular is independent of the amplitude of vibrations. We will denote this natural frequency by ω_1. When S is large and hence the amplitude of the response q very large, the system behaves like a linear system with the natural frequency now being determined by the slope of the two external segments of the $F(q)$ vs q curve. We will refer to the frequency of the linear system, with slope that of the external segments of the $F(q)$ curve, as ω_2.

Thus only for intermediate values of amplitude of the forcing function do we observe nonlinear phenomena. Here the natural frequencies for a nonforced undamped system lie between ω_1 and ω_2 and depend on the amplitude of the vibrations. This amplitude-frequency relationship is shown in Fig. 3 by the "backbone" curve. Throughout this paper the term "natural frequency range" will be used to denote the range from ω_1 to ω_2.

The principal nonlinear phenomenon occurring when the driving frequency is within the natural frequency range is the jump phenomenon. This is shown in Fig. 3, where the amplitude of q follows the curve AB, then jumps down to C and then on to D as frequency Ω of the forcing function is slowly increased. By slowly decreasing the frequency Ω the path followed is $DEFA$ with the jump occurring between E and F. The associated jumps in phase occurring at the same times also are shown in Fig. 3. The jump in phase is strikingly exhibited on the oscilloscope as shown in Fig. 4, where the response q is

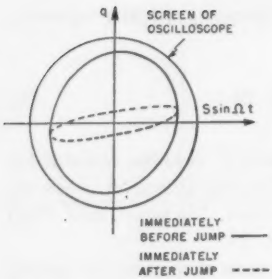

FIG. 4. Jump phenomenon in bilinear system as seen on an oscilloscope.

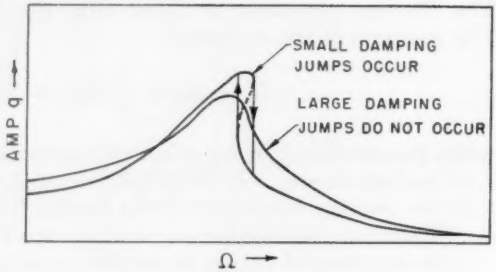

FIG. 5. Response curves for bilinear system showing the effect of damping.

plotted against the forcing function $S \sin \Omega t$. The dotted curve from B to E represents unstable solutions to the equations, none of which was observed in the experiment. An increase in damping has the effect of flattening the response curve and with sufficient damping the jump disappears, even for intermediate values of S, as shown in Fig. 5.

SUPERHARMONIC RESPONSES

When an integral multiple of the forcing frequency Ω was a frequency in the natural frequency range the response would consist of a superharmonic riding the fundamental response; for example, when the forcing frequency Ω was such that 3Ω was in the natural frequency range a third-order superharmonic would arise, the frequency of this third-order harmonic falling in the natural frequency range of the system. An interpretation of this phenomenon is that the sharp break in the restoring force curve was the impulse that, so to speak, "shock excited" these natural resonant frequency responses which appear as superharmonics of the forcing frequency. For the superharmonics from second to fifth order, the impulses occur fast enough to cause the harmonics to

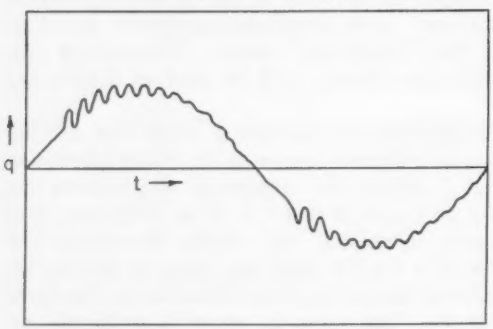

FIG. 6. Damped natural frequency appearing as superharmonic on the fundamental.

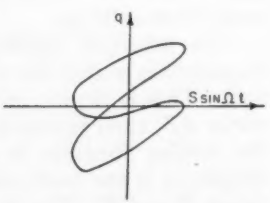

FIG. 7. Oscilloscopic trace showing combination of subharmonic and fundamental responses.

appear as stable responses superimposed on the fundamental. However, when the driving frequency becomes as low as 1/10 to 1/20 of the natural frequency range, the natural frequencies excited by the impulse have sufficient time to die out before the next impulse occurs, giving the appearance of a damped wave train at a natural frequency superimposed on the fundamental response at the forcing frequency (see Fig. 6).

SUBHARMONIC RESPONSES

When the frequency Ω of the forcing function was twice a frequency in the natural frequency range the system was capable of vibrating with a response containing a subharmonic component. When $\frac{\Omega}{2}$ did not lie in the "natural frequency range" the response frequency was the same as the forcing frequency. The relative magnitude of the subharmonic component with a frequency $\frac{\Omega}{2}$ associated with the fundamental component with frequency Ω was dependent upon the position of $\frac{\Omega}{2}$ in the "natural frequency range." This relative magnitude varied from zero to an almost pure subharmonic. See Fig. 7 which is a sketch of an oscilloscope trace of a response, the major part of which is a subharmonic.

Jump phenomena were observed associated with the subharmonic responses. These were best seen on the oscilloscope where the change in magnitude was associated with a marked phase change between the response and the forcing function.

SUMMARY AND CONCLUSIONS

The electronic analog of the bilinear system studied in this paper exhibited the usual fundamental response predicted for bilinear systems (6). For free vibrations a range of natural frequencies was found over which the frequency of vibration depended upon the displacement. Forced vibrations produced responses whose fundamental components confirmed the predicted behavior of such a system. Jump phenomena were associated with forced vibrations in the natural frequency range.

The excitation of responses containing harmonic components (superharmonics) whose frequencies lie in the natural frequency range, when the forcing frequency is below the natural frequency occurs through wide ranges of amplitude and frequency of the forcing function. The mechanism for their excitation seems to be the impulses that occur during each cycle due to the sharp breaks in the restoring force curve. Since the forcing frequency is lower than the natural frequency range the natural frequencies thus exhibited appear as harmonic components riding the fundamental response (see Fig. 6).

The excitation of subharmonics, that is responses which are sub-

multiples of the forcing frequency, depends upon the initial conditions and occurs for very limited ranges of forcing frequency and amplitude. The particular subharmonic produced was the one-half order subharmonic. When one half the driving frequency is in the natural frequency range and with an initial condition such that the displacement produces a restoring force beyond the sharp break in its curve, the phase relation between the driving frequency and the natural frequency of the system can be such that the impulse due to the sharp break in the restoring force curve occurs only once for each cycle of the forcing frequency. This impulse then sustains the natural frequency as a component of the response and is seen as a subharmonic response.

APPENDIX

Numerical Values of Circuit Parameters for a Particular "Run" of the Analog

The phenomena described in the foregoing pages occurred over wide ranges of numerical values of the circuit parameters. The natural frequency range has a lower limit of $\omega_1 = 100$ cps. and an upper limit $\omega_2 = 3000$ cps. The extent of the range ω_1–ω_2 was about 500 cps. for the higher frequencies and about 100 cps. for the lower frequencies. The numerical values of the circuit parameters for a particular "run" of the analog were these: The effective resistance R (see Eq. 1 and Fig. 2), which was due to losses in the inductor (etc.), was measured as 160 ohms. The capacitance C was 0.025 micro farads. The inductance L of the coil (General Radio Type 1490 B) was 1.0 henry. Referring to Fig. 1, the plot of $F(q)$ vs q, the slope of the central segment for this run was 1.35. The external segments had a slope of 2.8.

The function generator (see Fig. 2) utilized germanium diodes to produce the bilinear function $G(q)$. The diodes were connected in the feedback loop of a high gain d-c. amplifier. An isolating amplifier was incorporated in the function generator unit to avoid loading it, which would increase the effective value of R. The internal segment of the function $G(q)$ had a slope of 0.35 (for this particular run) while the external segments had a slope of 1.8. $F(q)$ of Eq. 1 is the sum $q + G(q)$.

For these values of the circuit parameters the natural frequency range was from $\omega_1 = 1160$ cps. to $\omega_2 = 1700$ cps.

The one-half order subharmonics which were obtained with these circuit parameters occurred when the driving frequency Ω ranged from 2400 to 3200 cps. The amplitude of the subharmonic component was found to be a function of both the amplitude of the driving function and its frequency. The investigation was not pursued further along these lines.

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RELIABLE CIRCUITS USING LESS RELIABLE RELAYS

BY

E. F. MOORE[1] AND C. E. SHANNON[1]

ABSTRACT

An investigation is made of relays whose reliability can be described in simple terms by means of probabilities. It is shown that by using a sufficiently large number of these relays in the proper manner, circuits can be built which are arbitrarily reliable, regardless of how unreliable the original relays are. Various properties of these circuits are elucidated.

Part I[2]

INTRODUCTION

In an important paper[3] von Neumann considers the problem of constructing reliable computing circuits by the redundant use of unreliable components. He studies several cases, one of which, for example, involves the construction of machines using as a basic component a "Sheffer stroke" organ.[4] Von Neumann shows that under certain conditions it is possible to combine a number of unreliable Sheffer stroke organs to obtain an element which acts like a Sheffer stroke organ of higher reliability. In fact, under certain conditions one can approach perfect operation by means of a sufficiently redundant circuit.

The present paper was inspired by von Neumann's work and carries out a similar analysis for relay circuits. It appears that relays are basically more adaptable to these error-correcting circuits than the neuron-like components studied by von Neumann. At any rate, our results go further than his in several directions.

In the first place, von Neumann needs to assume a certain fairly good reliability in his components in order to get started. With the Sheffer stroke organ, a probability of error less than 1/6 is absolutely necessary, and something like one in a hundred or better is required in the specific error-correcting circuits developed. The methods developed here, on the other hand, will apply to arbitrarily poor relays.

Secondly, the amount of redundancy required in our circuits for a

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[2] Part II will appear in this JOURNAL for October, 1956.

[3] J. VON NEUMANN, "Probabilistic Logics," California Institute of Technology, 1952. (Also Published in "Automata Studies," edited by C. E. Shannon and J. McCarthy, Princeton University Press, 1956.)

[4] The Sheffer stroke is the logical operation on two variables "not A and not B." It has the property that all logical functions can be generated in terms of it. A Sheffer stroke *organ* is a device with two binary inputs and one binary output which performs this logical operation. An unreliable component of this sort would give the proper output only with a certain probability.

given improvement in reliability is considerably different from that required by von Neumann. For example, in one numerical case that he considers, a redundancy of about 60,000 to 1 is required to obtain a certain improvement in operating reliability. The same improvement is obtained in relay circuits with a redundancy of only 100 to 1. We also show that in a certain sense some of our circuits are not far from minimal. Thus, in the numerical case just mentioned, our results show that a redundancy of at least 67 to 1 is necessary in any circuit of the type we consider. Hence, the actual circuits which achieve this improvement with a redundancy of 100 to 1 are not too inefficient in the use of components.

Another difference is that it is not necessary in the case of relays to use what von Neumann calls the "multiplexing system" in order to approach perfect operation on the final output. With his types of elements, the final output (without multiplexing) always has a definite residual unreliability. With the systems described here, this final probability of error can approach zero.

This paper is not intended for practical design purposes, but rather for theoretical and mathematical insight into the problem. There may, however, be some practical applications. The reliability of a commercial relay is typically very high, for example, one failure in 10^7 operations. However, there are cases where even this reliability is insufficient. In the first place, in large-scale computing machines an extremely large number of individual relay operations may be involved in one calculation, an error in any one of which could cause an error in the final result. Because of this, the Bell Telephone Laboratories' computers have made extensive use of self-checking and error-detecting schemes. A second type of situation requiring extreme reliability occurs when human safety is dependent on correct operation of a relay circuit, for example, railway interlocks, safety circuits on automatic elevators and in guided missiles, etc. It is possible that some of the simpler circuits we describe may be of some use in applications such as these. However, the results of this paper will not be directly applicable to actual relays which wear out with age, but only to idealized relays whose probability of failure are constant in time.

IDEALIZED RELAYS

We will prove results only for idealized relays whose failures can be described in one specific manner by means of probabilities. Their description allows only intermittent types of failures, and allows these only under the assumption that the probability of failure remains constant as time passes.

This idealization does not cover such actually possible cases as relays which wear out with age, relays whose windings burn out, or relays which have been wired into the circuit with an imperfect soldered

connection. It is also assumed that the circuit is not improperly designed or improperly wired and that there are no bits of solder to produce short circuits between different wires.

Since all of the above kinds of errors and failures can actually occur in practice, using real relays, the results of this paper do not strictly apply to such real relays. However, the two kinds of failures considered in this paper do actually occur in relays, so the kinds of circuits suggested are of some possible application.

The first kind of failure allowed is the failure of a relay contact to close, which in actual relays is often due to a particle of dust preventing electrical closure.

The second type of failure is the failure of a contact to open, which in actual relays is usually due to the welding action of the current passing through the contacts. We shall consider relay circuits in which the only causes of errors are of these two types—failure of contacts that should be closed to be actually closed and of contacts that should be open to be actually open. We will assume, in fact, that there are two probabilities associated with a contact on a relay. If the relay is

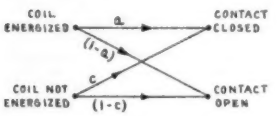


FIG. 1. Schematic representation of the transition probabilities.

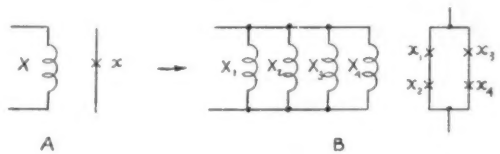


FIG. 2. One proposed way of transforming relay circuits to improve reliability.

energized, the contact is closed with probability a, open with probability $1 - a$. If the relay is not energized, the contact is closed with probability c and open with probability $1 - c$. If a is greater than c, we will call the contact a make contact; if a is less than c we call it a break contact. We assume that different contacts are statistically independent. With actual relays this is probably not too far from the truth for contacts on *different* relays and, indeed, this is all that is required for most of the results we wish to establish. In addition, we shall assume that on the successive times that a relay coil is energized its closures are statistically independent.

A relay of this type governed by probabilities a and c will be called a *crummy*[5] relay. Its probability operation may be represented schematically as in Fig. 1. This will be recognized as similar to diagrams used to represent a simple noisy communication channel, and indeed such a relay can be thought of as a noisy binary channel. The capacity of the corresponding channel will be zero if and only if $a = c$. We will

[5] "Crummy = crumby, esp. lousy," Webster's New International Dictionary. We chose the more modern spelling universally used in comic books.

see later that highly reliable computers can be constructed from a sufficient number of crummy relays if and only if $a \neq c$.

THE GENERAL METHOD OF IMPROVING RELIABILITY

In a general way the analysis we will give depends on constructing networks of contacts which act like a single contact but with greater reliability than the contacts of which they are composed. For example, in Fig. 2*A*, we have a crummy relay X with a make contact x. This relay might appear as a part of a large computing circuit. In Fig. 2*B* we replace this by four crummy relays X_1, X_2, X_3, X_4 whose coils in parallel replace the single coil X, and whose contacts are in the series parallel combination shown, this two-terminal circuit replacing the single previous x contact. If each of these four contacts has the probability p of being closed, it is easily seen that the probability of the four-contact circuit being closed is

$$h(p) = 1 - (1 - p^2)^2 = 2p^2 - p^4.$$

This function is plotted in Fig. 3. It will be seen that it lies above the diagonal line $y = p$ for p greater than 0.618 and lies below the line for

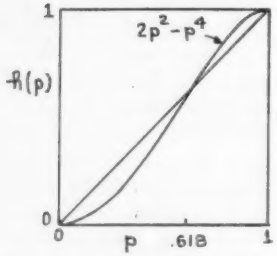
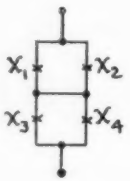


FIG. 3. The function describing the behavior of Fig. 2*B*.



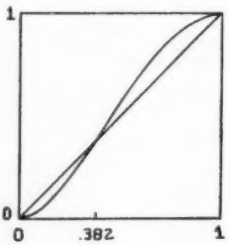


FIG. 4. Another series-parallel circuit and its associated function.

p less than 0.618. This means that if 0.618 is between the a and c of Fig. 1, Fig. 2*B* will act like a relay with better values of a and c, that is, values nearer to zero and one. For example, if the individual relays made errors with probabilities $1 - a = c = 0.01$, the circuit of Fig. 2*B* would make errors when the coils are energized with probability 0.000396, and when the coils are not energized with probability 0.0002. Thus a large improvement in reliability, both when the coil is energized and when it is not energized, is obtained by the use of this circuit.

Figure 4 shows another contact arrangement giving rise to a somewhat different function

$$h(p) = [1 - (1 - p)^2]^2 = 4p^2 - 4p^3 + p^4.$$

Here again, $h(p)$ is the probability of the network being closed, when the individual contacts each have probability p of being closed. The network of Fig. 4 is the dual of that in Fig. 2, and the curve is that obtained by interchanging 0 and 1 in both abscissa and ordinate in Fig. 3.

The bridge network of Fig. 5 gives rise to a symmetrical curve crossing the diagonal at $p = 0.5$. For this network we have:

$$h(p) = 2p^2 + 2p^3 - 5p^4 + 2p^5.$$

All of these networks tend to accentuate the nearness of p to its values 0 or 1 and thus tend to improve reliability. Many other networks have similar properties as we shall see. Furthermore, we will show that it is possible to find a network whose curve, Fig. 6, crosses the diagonal line for a value of p between any two given numbers a and c (no matter how close together) and in fact is less than δ at a and greater than $1 - \delta$ at c, for any positive δ. This means that an arbitrarily good relay can be made from a sufficient number of crummy relays.

It may be seen that this general procedure operates to improve the reliability of either make or break contacts. The only difference is the labeling of the points a and c.

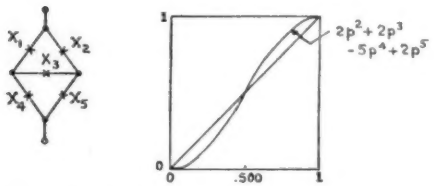


FIG. 5. A bridge circuit and its associated function.

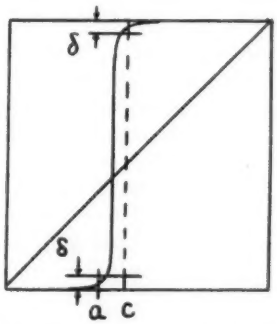


FIG. 6. The general form of curve of attainable functions.

PROPERTIES OF $h(p)$

Consider any two-terminal network made up of contacts each of which has a probability p of being closed. The network will have a probability, say $h(p)$, of being closed. We wish to investigate some of the properties of $h(p)$.

In the first place, $h(p)$ is a polynomial and may be written as follows:

$$h(p) = \sum_{n=0}^{m} A_n p^n (1 - p)^{m-n} \qquad (1)$$

where m is the total number of contacts in the network and A_n is the number of ways we can select a subset of n contacts in the network such that if these n contacts are closed, and the remaining contacts open, then the network will be closed. This is evident since (1) merely sums up the probabilities of the various disjoint ways that the network could be closed.

The first non-vanishing term in (1), say $A_s p^s (1-p)^{m-s}$, is related to the shortest paths through the network from one terminal to the other —s is the length of these paths and A_s the number of them. This is because in (1) all the elements of a subset which contribute to A_s must actually be on the path (otherwise A_s would not have been the first non-vanishing term). We will call s the *length* of the network. It is evident from (1) that near $p = 0$ the function $h(p)$ behaves as $A_s p^s$.

In a similar way, one can work with the probability of the network being open and write

$$1 - h(p) = \sum_{n=0}^{m} B_n (1-p)^n p^{m-n} \tag{2}$$

where B_n is the number of subsets of n contacts such that, if all contacts in a subset are open and the other contacts closed, the network is open. The first non-vanishing term in this series, say $B_t (1-p)^t p^{m-t}$, relates to the smallest cut sets of the network (sets of contacts which, if opened, open the network). Here t is the number of contacts in these minimal cut sets, and B_t the number of such cut sets. The reason is essentially as before. We will call t the *width* of the network. It is evident that, in the neighborhood of $p = 1$, $h(p)$ behaves as $1 - B_t (1-p)^t$.

The function $h(p)$ may also be calculated by other means. For example, fix attention on a particular contact in the network, N. Calculate the probability function for the network obtained from N by replacing this contact with a short circuit, say $f(p)$, and for the network obtained from N by replacing this contact with an open circuit, say $g(p)$. Then clearly,

$$h(p) = pf(p) + (1-p)g(p). \tag{3}$$

Furthermore we will have, whenever $0 \leq p \leq 1$,

$$f(p) \geq g(p). \tag{4}$$

This is intuitively evident since closing a connection certainly cannot decrease the probability of the network being closed. Formally, it follows from the relation (1), noting that the cases where the g network is closed are a subset of those in which f is closed, and consequently the terms in the expression for f dominate those in the expression for g.

If the network in question is planar, it will have a dual. Let $h_D(p)$ be the probability function for this dual network. For each

state of the contacts of the original network let us make correspond in the dual network the state in which corresponding contacts have the opposite value. Then states for which the original network is open correspond to states for which the dual network is closed. If the probability of closure of a contact in the dual network is $1 - p$, where p is the probability of closure in the original network, then the probabilities of corresponding states are equal. Consequently we will have

$$1 - h_D(1 - p) = h(p). \tag{6}$$

An example of this relation between the h functions for a network and its dual is given in Figs. 3 and 4. Either of these graphs can be obtained from the other by inverting, that is, by interchanging 0 and 1 in both abscissa and ordinate.

If the network is self-dual (for example the bridge of Fig. 5),

$$1 - h(1 - p) = h(p). \tag{7}$$

Substituting $p = 1/2$, we find $h(1/2) = 1/2$.

COMBINATION OF TWO NETWORKS

Consider now two networks N_1 and N_2 with functions $h_1(p)$ and $h_2(p)$. If N_1 and N_2 are connected in series, Fig. 7, the resulting net-

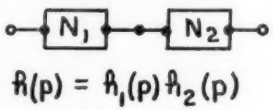


FIG. 7. Connection of two networks in series.

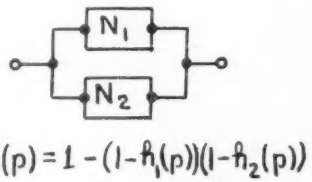


FIG. 8. Connection of two networks in parallel.

work will be closed only if both parts are closed. Hence, the resulting $h(p)$ function will be given by the product $h_1(p)\ h_2(p)$.

If N_1 and N_2 are connected in parallel, Fig. 8, the resulting network will be open only if both parts are open, an event with probability $(1 - h_1)(1 - h_2)$. Hence, the resulting $h(p)$ function for the parallel network will be $[1 - (1 - h_1)(1 - h_2)]$.

A third method of combining the two networks N_1 and N_2 is by "composition." By this we mean replacing each element of N_1 by a copy of N_2, as shown for a typical example by Fig. 9. It is evident that the composite network has an h function given by the composition of the two original h functions:

$$h(p) = h_1(h_2(p)). \tag{8}$$

If N_1 and N_2 are identical and this process is repeated $n - 1$ times, we obtain the n^{th} composition of h with itself, which we denote by

$$h^{(n)}(p) = h(h(h \cdots h(p) \cdots)).$$

The value of $h^{(n)}(p)$ can be found readily from the $h(p)$ curve by the staircase construction shown typically in Fig. 10 for $h^{(3)}(p_1)$. Thus, by composition, a greater improvement in reliability may be obtained with networks whose $h(p)$ curve crosses the diagonal but once. This effect, and the improvement by iteration relating to the staircase construction of Fig. 10, are very similar to situations in von Neumann's approach.

BOUNDS ON $h'(p)$

We will now deduce an interesting inequality concerning the slope of possible functions $h(p)$. As a corollary, we will show that any $h(p)$ function can cross the diagonal at most once.

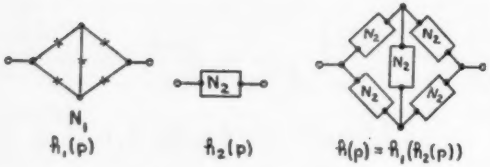


FIG. 9. Composition of two networks.

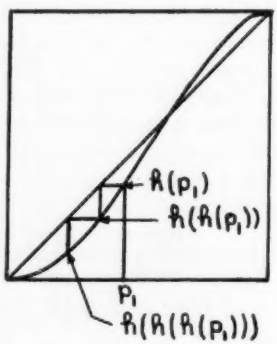


FIG. 10. The effect of iterated composition.

Theorem 1

$$\frac{h'(p)}{(1 - h(p))h(p)} > \frac{1}{(1 - p)p} \quad \text{whenever} \quad 0 < p < 1, \tag{9}$$

provided $h(p)$ is neither identically zero, identically one, nor identically equal to p.

This will be proved by an induction on the number of contacts in the network. We expand $h(p)$ as in (3) except that we expand it about some contact which lies on a path through the network, and then we assume that either the inequality holds for the functions f and g, or that they are among the three exceptional functions, and then we

prove the inequality for the function h. But since the contact actually lies on a path, the proof of (4) gives that $f(p) < g(p)$ for all p. Also we cannot have $1 - f(p) + g(p) = 0$ for any p, for if so, we would have $f(p) = 1$ and $g(p) = 0$, which implies there is no path through the network of g, and no cut set through the network of f, and hence $f(p) = 1$ and $g(p) = 0$ for all p, hence $h(p) = p$, contradicting the hypotheses of the theorem.

It can be seen that

$$(1 - p)p(f - g)(1 - f + g) > 0 \text{ whenever } 0 < p < 1, \quad (10)$$

since each of the terms is positive. Multiplying out,

$$pf - pg - pf^2 + 2pfg - pg^2 - p^2f + p^2g + p^2f^2 - 2p^2fg + p^2g^2 > 0.$$

Rearranging and factoring

$$-pf^2 + (1 - p)pf - (1 - p)g^2 - (1 - p)pg > \\ -[p^2f^2 + (1 - p)^2g^2 + (1 - p)2pfg].$$

Adding $pf + (1 - p)g$ to each side,

$$(1 - f)pf + (1 - p)pf + (1 - p)(1 - g)g - (1 - p)pg \\ > pf + (1 - p)g - [pf + (1 - p)g]^2 = h - h^2 = (1 - h)h. \quad (11)$$

Now, since by inductive assumption either $\frac{f'}{(1 - f)f} > \frac{1}{(1 - p)p}$, or we have one of the three exceptional functions, we have in any case that $(1 - f)f \leq (1 - p)pf'$ and similarly $(1 - g)g \leq (1 - p)pg'$. Using these in the left member of (11) we obtain

$$(1 - p)p^2f' + (1 - p)pf + (1 - p)^2pg' - (1 - p)pg > (1 - h)h.$$

Dividing by $(1 - p)p$,

$$pf' + f + (1 - p)g' - g > \frac{(1 - h)h}{(1 - p)p},$$

or

$$\frac{d}{dp}(pf + (1 - p)g) > \frac{(1 - h)h}{(1 - p)p},$$

$$\frac{h'}{(1 - h)h} > \frac{1}{(1 - p)p},$$

completing the proof.

If we replace the inequality (9) in the statement of the theorem

by an equality, that is if we set $\frac{y'}{(1-y)y} = \frac{1}{(1-p)p}$, we have a differential equation, the solutions of which form a one-parameter family of curves. The inequality (9) states that the permissible h functions corresponding to contact networks must have slopes greater than these y curves. If we solve this differential equation for the y curves we obtain

$$\frac{y(p)}{1-y(p)} = C\frac{p}{(1-p)}. \tag{12}$$

This family of curves is plotted in Fig. 11 for $C = 1/4, 1/3, 1/2, 1, 2, 3, 4$. Any possible $h(p)$ function must cross curves of this family with

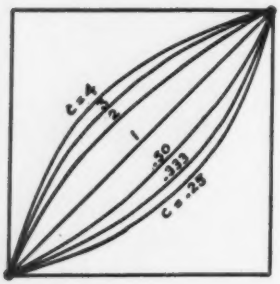


FIG. 11. The family of curves satisfying the equation

$$\frac{y(p)}{1-y(p)} = C\frac{p}{(1-p)},$$

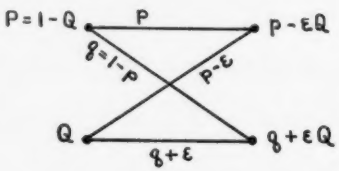


FIG. 12. A binary channel used to obtain an upper bound on the slope $h'(p)$.

a greater slope. Consequently, any $h(p)$ curve can cross one of these curves at most once in the open interval $0 < p < 1$. Since the straight line of slope 1 which goes through the origin is one member of this family, any $h(p)$ curve can cross this line once at most, say at the point $p = p_0$. Then applying the staircase construction as shown in Fig. 10, it can be seen that $h^{(n)}(p)$ approaches 0 as a limit for all $p < p_0$, and approaches 1 for all $p > p_0$. Thus any network whose $h(p)$ curve crosses this diagonal straight line can be composed with itself to obtain a network which improves reliability. In fact if we iterate the composition n times, we will have

$$\lim_{n\to\infty} h^{(n)}(p) = \begin{cases} 1 & p > p_0 \\ p_0 & p = p_0 \\ 0 & p < p_0 \end{cases}$$

where p_0 is the (necessarily unique) diagonal crossing point.

It is possible to place an *upper* bound on the slope $h'(p)$ by a curious

argument involving information theory. Consider the binary channel shown in Fig. 12. The rate of transmission for this channel will be

$$\begin{aligned} R &= H(y) - H_x(y) \\ &= -(p - \epsilon Q)\log(p - \epsilon Q) - (q + \epsilon Q)\log(q + \epsilon Q) \\ &\quad + (1 - Q)(p \log p + q \log q) \\ &\quad + Q[(p - \epsilon)\log(p - \epsilon) + (q + \epsilon)\log(q + \epsilon)]. \end{aligned}$$

For ϵ approaching zero, $(a + \epsilon)\log(a + \epsilon)$ is approximated by its Taylor series

$$a \log a + (1 + \log a)\epsilon + \frac{\epsilon^2}{a} + \cdots.$$

Using this in the above for all terms containing ϵ, we find that the constant terms and first order terms in ϵ vanish. The first non-vanishing terms are given by

$$R = (Q - Q^2)\frac{\epsilon^2}{pq} = \left[\frac{1}{4} - \left(Q - \frac{1}{2}\right)^2\right]\frac{\epsilon^2}{pq}.$$

It is evident from this last expression that R is maximized (when we vary Q) by $Q = 1/2$. This maximum R is, by definition, the channel capacity C. Thus as ϵ approaches zero in Fig. 12, the capacity C is asymptotic to $\frac{\epsilon^2}{4pq}$.

Now consider a crummy relay which has probability p of being closed when the relay is energized and $p - \epsilon$ of being closed when the coil is not energized. The relay may be thought of as a communication channel for which the coil is the input and the contact the output. If ϵ is very small, the capacity will be $\frac{\epsilon^2}{4pq}$. If we have n relays, with the same p and ϵ, the total capacity of this sytem, using the n coils as input and the n contacts as output, is $n\epsilon^2/4pq$, since the capacity of a set of independent channels is the sum of the individual capacities.

We wish to show from these capacity considerations that the probability function $h(p)$ for our contact networks must satisfy

$$\frac{dh}{dp} \leq \sqrt{\frac{n(1 - h)h}{(1 - p)p}}. \tag{13}$$

Consider a network N with n contacts and probability function $h(p)$. Let the individual relays and contacts have probabilities p_1 and ϵ as in Fig. 12. Then the network as a whole acts like a single relay with

parameters $h(p_1)$ and $h_2'(p_1)\epsilon$, (when ϵ is small). As such, it has a capacity $(h'\epsilon)^2/4(1-h)h$. This capacity must be less than or equal to that obtained when these n relays are used in the best possible way. Hence,

$$\frac{(h'\epsilon)^2}{4(1-h)h} \leq \frac{n\epsilon^2}{4(1-p_1)p_1}.$$

This being true for any p_1, we have, rearranging terms, the desired result

$$h' \leq \sqrt{\frac{n(1-h)h}{(1-p_1)p_1}}.$$

If this inequality is changed to an equality, we obtain the differential equation

$$\frac{\sqrt{n}\,dp}{\sqrt{(1-p)p}} = \frac{dh}{\sqrt{(1-h)h}}$$

the solution of which is

$$\sqrt{n}\sin^{-1}(1-2p) = \sin^{-1}(1-2h) + \theta. \tag{14}$$

For a given number of contacts n, a possible $h(p)$ curve must cross the corresponding family of curves (14) always with less or equal slope.

Another sort of upper bound on $h(p)$ functions obtained from n contacts can be found by a different argument. A two-terminal network corresponds to a Boolean function of the n contacts involved. However, it is not possible to realize all Boolean functions using only one make contact for each variable. Suppose we ignore these conditions of realizability and consider the class of all Boolean functions of n variables. For any such Boolean function there will be an $h(p)$ function, $h(p)$ being the probability that the function is equal to one if each variable has the (independent) probability p of being equal to one. Which Boolean functions have $h(p)$ functions with the greatest slopes and show the greatest sharpening effect on probabilities?

A Boolean function of n variables will be called a *quorum* function if there is some s, $0 \leq s \leq n$, such that if less than s of the variables are one the function is zero, and if more than s of the variables are one the function is one.

Theorem 2

If the h curve for any quorum function of n variables, say $h_Q(p)$, crosses the h curve of any other Boolean function of n variables, say $h(p)$, then at the point of crossing p_0 we have

$$h'(p_0) < h_Q'(p_0)$$

that is, the quorum function has the greater slope. Furthermore,

$$h(p) > h_Q(p) \qquad 0 < p < p_0$$
$$h(p) < h_Q(p) \qquad p_0 < p < 1.$$

This theorem says that, in a certain sense, the quorum functions are the best of all Boolean functions for our purposes of increasing reliability.

Proof: For any Boolean function of n variables, the $h(p)$ polynomial is made up of a sum of terms of the form p^iq^{n-i}, a term of this form for each state of the variables for which the Boolean function has the value one with i of the variables equal to one. A quorum function has the value one for all states with i less than s, say, and zero for all states with i greater than s. Hence the $h_Q(p)$ function is of the form

$$h_Q(p) = \sum_{i=0}^{s-1} \binom{n}{i} p^iq^{n-i} + Ap^sq^{n-s}. \quad 0 \leq A \leq \binom{n}{s}$$

Since h is not identical with h_Q but is equal in value to it at p_0, it follows that the h polynomial must miss some terms before (or at) i equals s and have some extra ones after (or at) i equals s. In other words, we can write

$$h(p) = \sum_{i=0}^{n} B_ip^iq^{n-i}$$

with $B_i \leq \binom{n}{i}$ Let $C(p) = \sum_{i=0}^{\tau} B_ip^iq^{n-i} + \alpha p^sq^{n-s}$ where α is B_s or A, whichever is smaller. Then we will have

$$h_Q(p) = C(p) + \sum_{i=0}^{\tau} D_ip^iq^{n-i}$$

$$h(p) = C(p) + \sum_{i=\tau+1}^{n} E_ip^iq^{n-i} \tag{15}$$

where the D_i and E_i are non-negative integers and τ is $s-1$ or s according as B_s or A was smaller.

Now we note that for an expression of the form $u(p) = p^iq^{n-i}$ we have

$$u'(p) = i\,p^{i-1}q^{n-i} - (n-i)p^iq^{n-i-1}$$
$$= \left(\frac{i}{p} - \frac{n-i}{q}\right) u(p) = \frac{i-pn}{pq}\,u(p).$$

Thus $\frac{u'}{u} = \frac{i - pn}{pq}$ is a monotone increasing function of i. Now all the terms in the sum in (15) for h_Q correspond to smaller values of i than those in the sum for h. If we let $u_Q(p)$ stand for any term in the sum in h_Q and $u(p)$ stand for any term in the sum in h, we will have

$$\frac{u_Q'}{u_Q} < \frac{u'}{u}$$

and hence there will exist a constant K such that

$$\frac{u_Q'}{u_Q} < K < \frac{u'}{u},$$

and

$$u_Q' < K u_Q, \qquad K u < u'.$$

Summing the first inequality over all the different terms u_Q, and the second over all the u, we obtain

$$\sum u_Q' < K \sum u_Q, \qquad K \sum u < \sum u'.$$

But evaluating at p_0, we have $\sum u_Q = \sum u$, and consequently

$$\sum u_Q' = \sum u',$$
$$h_Q'(p_0) < h'(p_0).$$

The remainder of the theorem follows readily by noting that to contradict it, since the h and h_Q curves are continuous, would require that they cross at a point different from p_0 and in such a way as to contradict the first part of the theorem.

NETWORKS OF A GIVEN LENGTH AND WIDTH

We have seen that the orders of flatness of $h(p)$ in the neighborhoods of $p = 0$ and $p = 1$ are related to the "length" and "width" of the network in question. It is clear that in the case of practical importance, the values of p of interest will be in these neighborhoods, that is, the relays will be initially quite reliable. In this section we will develop some results relating these orders of flatness with the number of elements in the network.

Theorem 3

If a network N has length l and width w it contains at least lw contacts. Equivalently, if $h(p)$ behaves like Ap^l near $p = 0$, and if $1-h(p)$ behaves like $B(1 - p)^w$ near $p = 1$, the corresponding network contains at least lw contacts.

Proof: We associate an integer with each contact in N by the following process. Contacts directly connected to the left terminal of N are labeled "1," contacts connected to those labeled 1 but not already labeled are numbered "2," and so on inductively. In general, a contact will be labeled n if it is possible to find a path to the left terminal through $n - 1$ other contacts but there is no such path through a smaller number.

The set of contacts labeled n for any particular n from 1 to l will be shown to form a cut set of the network. This is true since every path through the network starts at the left terminal with a contact labeled 1 and ends at the right terminal with a contact labeled l or more (if any of the contacts touching the right terminal were labeled with numbers less than l the length of N would be less than l). Along any path, the numbering changes by 0 or ± 1 in going from one contact to the next. Hence every path in going from contacts numbered

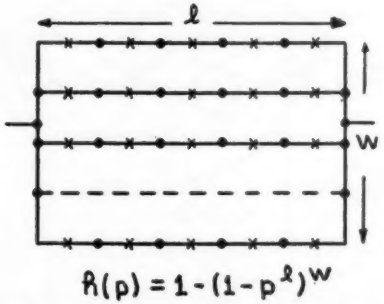


FIG. 13. A series-parallel network of length l and width w.

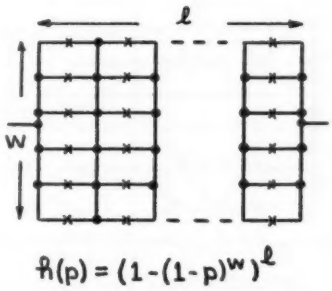


FIG. 14. Another series-parallel network of length l and width w.

1 to those with numbers $\geq l$ must pass through every intermediate value. Consequently if all contacts labeled n (for $1 \leq n \leq l$) are deleted from N, all paths are broken and these contacts thus form a cut set.

Since the network is of width w, every cut set contains at least w contacts. Thus there are at least w contacts labeled 1, at least w labeled 2, $\cdots$, and at least w labeled l. The network therefore contains at least wl contacts.

The alternative statement of Theorem 3 follows from remarks made in connection with Eqs. 1 and 2.

It is possible to achieve the "dimensions" l and w with *exactly* lw contacts in a wide variety of ways. For example, we can make a series chain of l contacts and parallel w copies of this (Fig. 13). Dually, w contacts can be paralleled and l copies of this placed in series (Fig. 14).

Theorem 4

A complete characterization of minimal networks with dimensions l and w is the following. Let Y and Z be the terminal nodes, s_0 be the set consisting of Y alone, and s_l be the set consisting of Z alone. In addition to s_0 and s_l there will be $l-1$ subsets of nodes $s_1, s_2, \cdots, s_{l-1}$. There will be precisely w elements connecting nodes in s_n to nodes in s_{n+1} $(n = 0, 1, \cdots, l-1)$. Finally, if any node in s_j has m elements connecting it to nodes in s_{j-1}, then it has m elements connected to nodes in s_{j+1} $(j = 1, 2, \cdots, l-1)$.

This means that any such minimal network with dimensions l and w can be obtained from the network of Fig. 13 by making appropriate connections among nodes in the same vertical line. When all the nodes in each vertical line are connected together, for example, the result is Fig. 14. Another possibility is shown in Fig. 15.

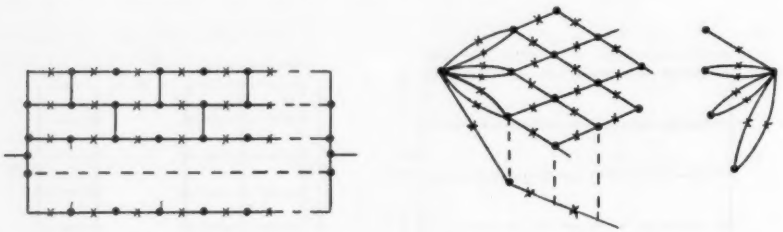


FIG. 15. A hammock network of length l and width w.

To show that any minimal lw network is of the form described in Theorem 4, first note that in our preceding proof, each of the numbered cut sets must contain precisely w elements, and these elements must run between elements of lower numbers and higher numbers. The nodes between elements numbered $j-1$ and j will belong to subset s_j in the above characterization. Now suppose that some node in s_j has m elements going to nodes in s_{j-1} and $m+p$ going to nodes in $s_{j+1}(p > 0)$. The elements numbered $j+1$ form a cut set of w elements. It is easily seen that if the $m+p$ members of this, going from the node in question, are replaced by the m elements going to nodes in s_{j-1}, then we will still have a cut set but one with less than w elements, a contradiction. Consequently any minimal network of dimensions l and w is of the type described in our characterization.

To show the converse, that any network of the type characterized has dimensions l and w, note first that to go from one terminal to the other the path must pass through nodes belonging to $s_1, s_2, \cdots, s_{l-1}$. Hence any path is of length at least l and the network is of length l. Now consider any cut set c. We will show that c contains at least w

elements. Consider the smallest-numbered contacts of c. Suppose one of these is connected from node A in s_{j-1} to node B in s_j. Then either all elements from B to nodes in s_{j-1} are in the cut set or the one in question is not essential to the cut set and may be eliminated, giving a still smaller cut set. In the former case, this group of elements can be replaced by an equal number, those going from node B to members of s_{j+1}, preserving the cut set property. Proceeding in this way, the cut set is gradually worked over toward the right-hand terminal, either reducing or keeping constant the number of elements in the cut set.

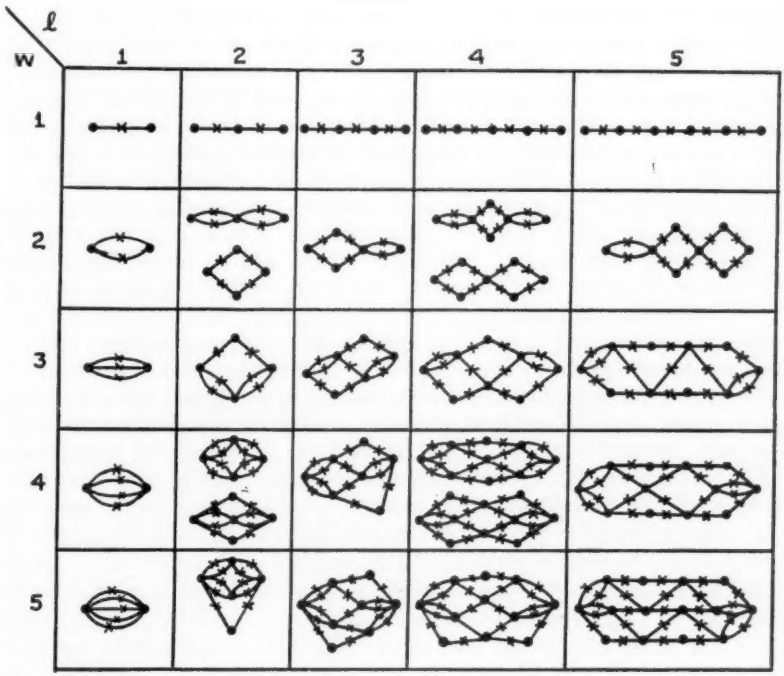


FIG. 16. Hammock networks of various lengths and widths.

When all the elements of the cut set are adjacent to the right-hand terminal there are exactly w members. Consequently there were at least that many in the original cut set, as we wished to prove.

An interesting type of minimal lw network is obtained by putting alternate connections in Fig. 13, leading to the brick-wall appearance of Fig. 15A. When redrawn, pulling together the vertical connections, the network appears as in Fig. 15B, and we will call networks of this type hammock networks. Figure 16 shows some of the simple cases

of hammock networks. It will be seen that if both l and w are even, there are two possible hammock networks with these dimensions. If either or both are odd, there is only one. Furthermore, the dual of a hammock network with length l and width w is a hammock network with length w and width l. These hammock networks are, in a sense, midway between the extreme minimal lw networks of Figs. 13 and 14, having half of the connections required to go from Fig. 13 to Fig. 14. In the case where l and w are equal and odd the (unique) hammock network is self-dual.

(To be continued)

THE POTENTIAL ENERGY OF A SMALL RIGID BODY IN THE GRAVITATIONAL FIELD OF AN OBLATE SPHEROID

BY

R. E. ROBERSON[1] AND D. TATISTCHEFF[2]

INTRODUCTION

One of the elementary problems of particle dynamics is the motion of a point mass in the field of a homogeneous gravitating sphere, that is, in an inverse square field. The usual illustrations of the theory are the motion of a small body in the field of the earth, of the satellites of the planets, or of the motion of the planets themselves in the field of the sun. However, in some of these cases the gravitating body is close to an oblate spheroid, and there exist physical instances in which the oblateness is significant. For example, Brouwer (1)[3] and others have treated the motion of a small body in the field of an oblate spheroid with applications to the moons of Jupiter. In the case of the earth, oblateness is also important to the motion of the moon; or, one could envision its effect as a perturbation of the motion of a projectile.

An obvious generalization of this problem is to the case of a distributed rigid body in the field of an ellipsoid. Physical problems also exist in which the shape of the subsidiary body is important to a complete description of the motion, as in the problem of close binary stars mentioned by Brouwer. (Although, as Zagar (5) observes, deformability of the bodies plays an essential role in this problem.) The purpose of this note is to give the form of the mutual potential function for some rigid bodies of small size relative to the principal gravitating body. This represents the first step in the treatment of the motions of such pairs of bodies in various special cases.[4] It can be considered a generalization of the extended MacCullagh formula (3, p. 54), together with an application of the result to several special shapes of the subsidiary body.

GENERAL POTENTIAL FUNCTION

Let P_1 be the center of the principal gravitating body and P_2 be a field point located relative to P_1 by $\bar{R}$. Suppose that D is a small

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

[4] EDITOR'S NOTE: An application of the results of this paper which is of some current interest is to the motion of (and body torque on) a close artificial satellite of the earth.

rigid body with center of mass at R_0, define $\bar{r} = \bar{R} - \bar{R}_0$, and let $\rho(\bar{r})$ be the density of D. The mutual potential of the bodies is

$$U(\bar{R}_0) = \iiint_D f(\bar{R}_0 + \bar{r})\rho(\bar{r})d\tau \tag{1}$$

where $f(\bar{R})$ is the potential function of the principal body per unit mass as the field point, and $d\tau$ is a volume element. It is well known (2) that for a rectangular cartesian coordinate system

$$f(\bar{R}_0 + \bar{r}) = f(\bar{R}_0) + \bar{r}\cdot\nabla f(\bar{R})\Big|_{\bar{R}_0} + \tfrac{1}{2}(\bar{r}\cdot\nabla)^2 f(\bar{R})\Big|_{\bar{R}_0} + \cdots. \tag{2}$$

The meaning of "small" in the statement that D is a small but not infinitesimal body is that only the lowest powers of $\bar{r}$ which appear when Eq. 2 is used in Eq. 1 are retained. Making this substitution and recognizing that $\iiint F\rho(\bar{r})d\tau = 0$ because $\bar{R}_0$ is the center of mass, one obtains the approximate result

$$U(\bar{R}_0) = mf(\bar{R}_0) + \tfrac{1}{2}\nabla\cdot\left(\iiint_D \bar{r}\bar{r}\rho(\bar{r})d\tau\right)\cdot\nabla f(\bar{R})\Big|_{\bar{R}_0} \tag{3}$$

where m is the total mass of D.

The triple integral of Eq. 3 will be denoted by I^*. It can be expressed easily in terms of inertia tensor components relative to the $\bar{e}_x$, $\bar{e}_y$, $\bar{e}_z$ axes, as usually defined (for example, in (4), p. 227).

$$\begin{aligned} I^* = {} & \tfrac{1}{2}(I_{yy} + I_{zz} - I_{xx})\bar{e}_x\bar{e}_x - I_{xy}\bar{e}_x\bar{e}_y - I_{xz}\bar{e}_x\bar{e}_z \\ & - I_{yz}\bar{e}_y\bar{e}_z + \tfrac{1}{2}(I_{zz} + I_{xx} - I_{yy})\bar{e}_y\bar{e}_y - I_{yz}\bar{e}_y\bar{e}_z \\ & - I_{xz}\bar{e}_x\bar{e}_z - I_{xy}\bar{e}_x\bar{e}_y + \tfrac{1}{2}(I_{xx} + I_{yy} - I_{zz})\bar{e}_z\bar{e}_z. \end{aligned} \tag{4}$$

In the sequel, attention will be focused on

$$U_1 = \tfrac{1}{2}\nabla\cdot I^*\cdot\nabla f(\bar{R})\Big|_{\bar{R}_0}. \tag{5}$$

The remaining term U_0 in U is a potential function for the smaller body as if it were a point mass, and does not require further discussion.

Thus far, nothing has restricted the shape of either body. That of the principal body is implicit in $f(\bar{R}_0)$, and that of the small body is characterized by I^*. Before introducing a particular potential function $f(\bar{R})$, it is convenient to represent I^* explicitly in terms of the

subsidiary body's geometry (represented by principal moments of inertia I_1, I_2, I_3) and a set of Euler angles giving the orientation of D relative to the large body.

Let $\bar{e}_1$, $\bar{e}_2$, $\bar{e}_3$ be unit vectors embedded in the large body, and let the position vector of the small body be characterized by a radial distance R_0, a latitude angle λ_0, and a longitude angle Λ_0 as shown in Fig. 1. Let unit vectors $\bar{e}_x$, $\bar{e}_y$, $\bar{e}_z$, also shown in Fig. 2, be in the directions of increasing Λ, λ, and R, respectively, and suppose that the principal axes $\bar{e}_1'$, $\bar{e}_2'$, and $\bar{e}_3'$ of the small body are related to $\bar{e}_x$, $\bar{e}_y$, $\bar{e}_z$ as shown in the unit sphere diagram of Fig. 2.

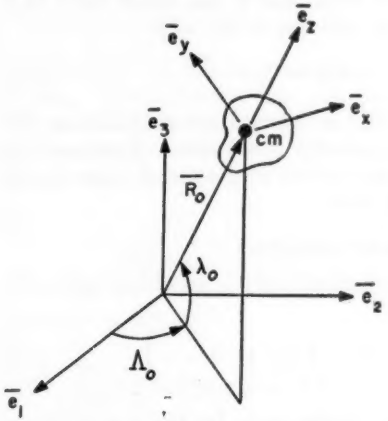

FIG. 1. Position variables for small body relative to large body.

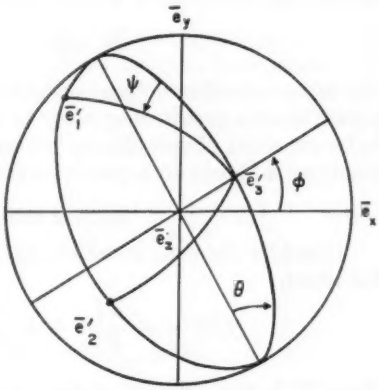

FIG. 2. Relationship of $\bar{e}_1'$, $\bar{e}_2'$, $\bar{e}_3'$ to $\bar{e}_x$, $\bar{e}_y$, $\bar{e}_z$.

Denoting by I_1, I_2, I_3 the principal moments of inertia of the smaller body about $\bar{e}_1'$, $\bar{e}_2'$, $\bar{e}_3'$ and using the transformation matrix between this set of base vector and the set $\bar{e}_x$, $\bar{e}_y$, $\bar{e}_z$, it follows that

$$\begin{aligned} I_{xx} = {} & I_1(\cos\psi \sin\varphi + \sin\psi \cos\varphi \cos\theta)^2 \\ & + I_2(\sin\psi \sin\varphi - \cos\psi \cos\varphi \cos\theta)^2 + I_3 \cos^2\varphi \sin^2\theta \end{aligned} \tag{6}$$

$$\begin{aligned} I_{xy} = {} & -I_1(\cos\psi \sin\varphi + \sin\psi \cos\varphi \cos\theta)(\cos\psi \cos\varphi - \sin\psi \sin\varphi \cos\theta) \\ & - I_2(\sin\psi \sin\varphi - \cos\psi \cos\varphi \cos\theta)(\sin\psi \cos\varphi + \cos\psi \sin\varphi \cos\theta) \\ & + I_3 \sin^2\theta \sin\varphi \cos\varphi \end{aligned} \tag{7}$$

$$\begin{aligned} I_{xz} = {} & -I_1(\cos\psi \sin\varphi + \sin\psi \cos\theta) \sin\psi \sin\theta \\ & + I_2(\sin\psi \sin\varphi - \cos\psi \cos\varphi \cos\theta) \cos\psi \sin\theta \\ & + I_3 \sin\theta \cos\theta \cos\varphi \end{aligned} \tag{8}$$

$$I_{yy} = I_1(\cos\psi\cos\varphi - \sin\psi\sin\varphi\cos\theta)^2 + I_2(\sin\psi\cos\varphi + \cos\psi\sin\varphi\cos\theta)^2 + I_3\sin^2\varphi\sin\theta \tag{9}$$

$$I_{yz} = I_1(\cos\psi\cos\varphi - \sin\psi\sin\varphi\cos\theta)\sin\psi\sin\theta - I_2(\sin\psi\cos\varphi + \cos\psi\sin\varphi\cos\theta)\cos\psi\sin\theta + I_3\sin\varphi\cos\theta\sin\theta \tag{10}$$

$$I_{zz} = I_1\sin^2\psi\sin^2\theta + I_2\cos^2\psi\sin^2\theta + I_3\cos^2\theta. \tag{11}$$

There is no advantage in substituting these results into Eq. 4 at this point. However, it is of interest to note that if the small body is a sphere, the products of inertia vanish and one is left with

$$2U_1(\bar{R}_0) = \nabla^2 f(\bar{R}_0) = 0, \tag{12}$$

the latter equality following from the interpretation of $f(\bar{R}_0)$ as the potential of a gravitating body at a point in free space. This result is to be expected physically, for no torque about its center of mass should result on a sphere in a gravitational field.

FIELD OF AN OBLATE SPHEROID

Consider the case in which the principal body is an oblate spheroid, for which

$$f(\bar{R}) = -\frac{K}{R}\left[1 + \mu\frac{a^2}{R^2}(1 - 3\sin^2\lambda) + \cdots\right]. \tag{13}$$

Here K is the product of the mass of the body by the gravitational constant, a is a constant with the dimensions of length which may conveniently be taken to be any principal radius of the large body, and μ is a dimensionless form factor of the order of the ellipticity of the oblate spheroid. Terms involving powers of $(a/R)^2$ higher than the first are omitted from Eq. 13, although they can be added easily if desired. The constant μ has a simple interpretation in terms of principal moments of inertia of the main body if that body is homogeneous. (For example, see (4), p. 402.)

In using Eq. 13 in Eq. 5, use

$$\nabla = \bar{e}_x\frac{\partial}{\partial_x} + \bar{e}_y\frac{\partial}{\partial_y} + \bar{e}_z\frac{\partial}{\partial_z} \tag{14}$$

where x, y, z are position coordinates relative to $\bar{e}_x$, $\bar{e}_y$, $\bar{e}_z$. Also, recognize that in Eq. 13

$$R = \sqrt{(R_0 + z)^2 + x^2 + y^2} \tag{15}$$

$$R\sin\lambda = (R_0 + z)\sin\lambda_0 + y\cos\lambda_0. \tag{16}$$

Performing the differentiation indicated by Eq. 5, one obtains

$$U_1 = \frac{K}{4R_0^3}\left[1 + 3\mu\frac{a^2}{R_0^2}(1 - 5\sin^2\lambda_0)\right][I_{yy} + I_{zz} - I_{xx}]$$
$$+ \frac{K}{4R_0^3}\left[1 + 3\mu\frac{a^2}{R_0^2}(3 - 7\sin^2\lambda_0)\right][I_{zz} + I_{xx} - I_{yy}]$$
$$- \frac{K}{2R_0^3}\left[1 + 3\mu\frac{a^2}{R_0^2}(2 - 11\sin^2\lambda_0)\right][I_{xx} + I_{yy} - I_{zz}]$$
$$+ \frac{12\mu Ka^2}{R_0^5} I_{yz}\sin 2\lambda_0. \qquad (17)$$

Equation 17 can be rewritten in terms of principal moments of inertia. First, observe that any portion of U_1 which is independent of the rotational aspect of the small body properly should be split out and combined with U_0 to describe the motion of the center of mass. Specifically, transfer to U_0 in this way the quantity

$$-\frac{K}{2R_0^3}\left[1 + 3\mu\frac{a^2}{R_0^2}(1 - 10\sin^2\lambda_0)\right][I_1 + I_2 + I_3]. \qquad (18)$$

Then one is left with

$$U_1 = \frac{3K}{2R_0^3}[I_1\sin^2\psi\sin^2\theta + I_2\cos^2\psi\sin^2\theta + I_3\cos^2\theta]$$
$$+ \frac{3\mu Ka^2}{2R_0^5}[(5\text{–}27\sin^2\lambda_0)I_{xx} - 2\cos^2\lambda_0 I_{yy} + 8\sin 2\lambda_0 I_{yz}]. \qquad (19)$$

The I_{xx}, I_{yy}, I_{yz} given by Eqs. 9, 10, and 11 are to be substituted into the μ-term of Eq. 19, but there is no advantage in writing out the result in full.

The results at this point are analogous to those of Zagar (**6**) for the attraction of spheroids, except that he has expanded the results in powers of the eccentricities of the two bodies.

SOME SPECIAL RESULTS

By setting $\mu = 0$ in Eq. 19, one finds the potential function for a small non-spherical body in the field of a large gravitating sphere. This function will give rise to torques on the small body, but note that no torque will appear about the axis of φ. Physically, this means that the torques on such a body depend only on the deviation of its "vertical" ($\bar{e}_3'$) axis from the geocentric radius through its center of mass, and

the rotation angle of the body about this vertical axis. No torque appears to tend to change the direction of the plane of $\bar{e}_3'$ and $\bar{e}_z$. This is to be expected, for a spherical main body has $\bar{e}_z$ as an axis of symmetry.

It is a corollary that the "spring constant" for φ rotation torques is of the order of μ times those for θ and ψ rotation torques.

Another special case of interest is that of a small body with circular symmetry about the $\bar{e}_3'$ axis, $I_1 = I_2 = I \neq I_3$. In this case, Eq. 19 reduces to

$$U_1 = \frac{3K}{2R_0^3}(I - I_3)\sin^2\theta + \frac{3\mu Ka^2}{2R_0^5}(I - I_3)\sin\theta[(17-29\sin^2\lambda_0)\sin\theta - 8\sin 2\lambda_0 \sin\varphi\cos\theta]. \quad (20)$$

The dominant term in the expression (for small μ) involves only the angle θ. Note that $\theta = 0$ is a position of stable equilibrium only if $I > I_3$. The potential is a function of φ only through terms of the order of μ, so that the corresponding spring constant is very weak for the case of small oblateness.

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NOTES FROM THE NATIONAL BUREAU OF STANDARDS

LOW TEMPERATURE STORAGE OF FREE RADICALS

The National Bureau of Standards has developed a technique for capturing and storing large numbers of highly reactive molecular fragments at temperatures near absolute zero. By this method, unstable atoms and free radicals, known to exist but momentarily in flames and hot gases, are produced in an electric discharge, frozen into immobility, and trapped in solid form. Because these atoms are frozen in the excited state, they can be conveniently studied by optical spectroscopy.

In experiments to date the Bureau has produced solids containing atomic nitrogen and oxygen, and possibly atomic hydrogen and an unstable hydroxy (OH) molecule. These solids have very unusual properties, emitting bright glows, blue "flames," and colored flashes of light. When warmed 20 or 30 deg., they combine very actively, releasing large quantities of stored energy, principally as heat. Other possible fields of application include solid state physics and basic chemistry. Here the trapped atoms could be used as powerful probes into the solids containing them. From a study of their properties, information could be obtained about the arrangement of the atoms and molecules in the solid and about the forces acting on them. Similarly the mechanism of diffusion of atoms and of reactions between atoms and molecules could be studied.

These experiments were begun at NBS in 1954 by H. P. Broida and J. R. Pellam[1] and are being continued by H. P. Broida, A. Bass, and O. Lutes[2] of the Bureau's temperature measurements laboratory. C. M. Herzfeld of NBS is carrying out theoretical investigations[3] on the systems. The research is supported chiefly by the Office of Naval Research and the U. S. Air Force through the Office of Scientific Research of the Air Research and Development Command.

Within the last five years, several methods have been developed in other laboratories for stabilizing free radicals at low temperatures. However, the present technique has the advantage that the free radicals are stored in highly excited states as a result of the electric discharge, making it possible to study and analyze them by spectroscopic techniques. Also, since the radicals are collected at much lower tem-

[1] H. P. Broida and J. R. Pellam, *Phys. Rev.*, Vol. 95 p., 845 (1954); H. P. Broida and J. R. Pellam, *J. Chem. Phys.*, Vol. 23, p. 409 (1955).

[2] A. M. Bass and H. P. Broida, *Phys. Rev.*, Vol. 101, p. 1740 (1956); H. P. Broida and O. S. Lutes, *J. Chem. Phys.*, Vol. 24, p. 484 (1956).

[3] C. M. Herzfeld and H. P. Broida, *Phys. Rev.*, Vol. 101, p. 606 (1955).

peratures than in previous methods, they can be stored longer in the uncombined form.

In this method, gases containing molecules of nitrogen, hydrogen, oxygen, or water are first passed through a high-frequency electric discharge, then frozen very suddenly at 4.2° K., just a few degrees above absolute zero.

The discharge is maintained in a waveguide resonator by a 2450-Mc. pow r supply. From the resonator, a glass tube leads the resulting molecular fragments into an evacuated metal vessel containing a cold surface in contact with a liquid helium bath. To prevent solidification of the discharge products at temperatures above 4.2° K., the gases are carried to the cold surface through a passageway that is kept near room temperature by contact with warm helium gas. When the gases reach the cold surface, they freeze into solid form there.

Various techniques can then be used to study the resulting solids. Through windows in the metal vessel, a number of spectrographs of different types can be aimed at the cold surface, and the light given off by the solids frozen on the surface can be analyzed. To study the light absorbed by the solids, the gases are condensed on a transparent cold surface, and light is passed through the windows of the vessel, the condensed material, the cold surface, and finally into the spectroscopes. To determine the heat evolved by recombination of atoms, the gases are condensed into a small, simple low-temperature calorimeter, and the heat evolved by the material as it warms up is measured.

Because the free radicals produced by this method can be kept unchanged for many hours, a more detailed study of their properties has been possible than in previous work. For example, during the discharge through nitrogen the condensed solid emits a bright green glow, which tends to become yellow at high flow rates. The spectra obtained from this glow show that the structure of the solid condensed from the discharge differs from that of ordinary solid nitrogen. In fact, each atom of the condensed nitrogen forms a very loosely bound complex with some neighboring molecule, and this complex has properties that differ from those of the separate atom and molecule. The evidence shows that the complex is a definite unit. By analyzing its spectra, the forces holding it together can be studied.

Brilliant blue flashes are also observed from the solid surface during the nitrogen discharge. These flashes are thought to be due to local warming. After the flow of nitrogen is stopped, the discharge goes out and a green afterglow from the cold collected material persists for several minutes, decaying with a half-life of about 15 seconds. When the green afterglow has disappeared, sudden warming of the vessel walls (to between 25° and 35° K.) causes a blue "flame" which appears to "burn" through the condensed material. The light and heat thus given off are evidently due to recombination of the atoms into molecules.

When the light is analyzed spectroscopically, it gives information about the structure of the molecule just formed and about the forces acting between two atoms. The reaction of a nitrogen atom with an oxygen atom can also be studied in this way.

When oxygen is passed through the discharge, a clear, glassy deposit is obtained which has rather complex absorption spectra. Upon warming this material to about 20° K., the original deposit evaporates and a solid violet material condenses on the surface. This substance has been identified as a mixture of oxygen and ozone. Further warming of the violet substance produces ozone in large quantities. Initiation of chemical reactions in this way, by warming the frozen material, indicates the possibility of opening up a new field of very low temperature chemistry. Because of the high chemical activity of the free radicals, new chemical compounds might be formed by this process.

Low-temperature condensates have also been obtained from hydrogen and water. While the solids formed show complex behavior similar to that of nitrogen and oxygen, the results are still incompletely understood.

LOW-TEMPERATURE ALIGNMENT OF RADIOACTIVE NUCLEI

Low-temperature research at the National Bureau of Standards has succeeded in aligning the nuclei of three radioactive elements—cerium-139, cerium-141, and neodymium-147. These results were achieved by cooling samples of the three materials to within a few thousandths of a degree of absolute zero. At such temperatures the effects of thermal agitation become so small that atomic nuclei can line up in a given direction within the crystal lattice. A corresponding directional effect can then be observed in the emitted radiation.

The nuclear alignment experiments[1] were carried out by Drs. E. Ambler and R. P. Hudson of the Bureau staff in cooperation with Dr. G. M. Temmer of the Carnegie Institution of Washington. Initial phases of the work were sponsored by the Office of Naval Research.

Low-temperature alignment of nuclei promises to provide a new tool for studying the processes of nuclear disintegration. The nucleus may be regarded as a magnetic top spinning about an axis. If this spinning magnet is radioactive, the orientation of the spin axis will determine the directions in which the nucleus emits radiation. Normally, when nuclei are randomly oriented, a radioactive specimen emits gamma rays with equal intensity in all directions. However, when the nuclei are aligned, the intensity of gamma radiation varies with angle of emission. By measuring the degree of this directional effect, valuable

[1] For further details, see "Alignment of Cerium-141 and Neodymium-147 Nuclei," by E. Ambler, R. P. Hudson, and G. M. Temmer, *Phys. Rev.*, Vol. 97, p. 1212 (1955); and "Alignment of Three Odd-A Rare Earth Nuclei," the by same authors, *ibid.*, Vol. 101, p. 1096 (1956).

information can be obtained concerning the decay scheme of the nuclei, and an insight can be gained into the mechanisms controlling such processes. For example, the magnetic moment of the nucleus can be determined as well as the changes in angular momentum accompanying the emission.

In the Bureau's experiments, radioactive nuclei were incorporated into certain inorganic crystals formed by the elements studied, which were then cooled to temperatures as low as 0.003° K. Nuclear alignment was observed by measuring the angular distribution of the intensity of the gamma radiation emitted by the crystals.

Inasmuch as the crystals used were paramagnetic, the necessary low temperatures could be conveniently produced by the method of adiabatic demagnetization. In this method a paramagnetic crystal is first magnetized by a powerful magnet. The resultant heat of magnetization produced in the crystal is removed from the system. Then when the magnetic field is turned off, the reverse effect occurs, and the temperature of the crystal falls to a very low value. The specimen soon begins to reheat, of course, but if due care has been taken to reduce heat leaks, the rate of heating is sufficiently slow to allow enough time for measurements.

In the Bureau's experiments a magnetic field of about 23,000 oersteds was used. A radioactive crystal containing the element under study was mounted on a thermally insulating support within a glass tube containing a small amount of "exchange gas" (helium at low pressure). The exchange gas provided thermal contact between the crystal and a surrounding bath of liquid helium boiling at about 1° K. under reduced pressure. The liquid helium bath was protected against heat influxes by a Dewar vessel, which was in turn surrounded by liquid nitrogen.

When the magnet was switched on, the heat of magnetization was conducted from the crystal through the exchange gas to the liquid helium. This kept the temperature of the crystal from rising. The crystal was then isolated thermally by pumping the exchange gas away. Thus, when the magnet was turned off, the temperature of the crystal fell appreciably.

To observe nuclear alignment, the apparatus was then quickly moved into position between two scintillation counters, and the intensity of gamma radiation was measured along two different directions. As the crystal warmed up, a gradual decrease in the degree of alignment was observed. Finally, when the temperature reached 1° K., the nuclei were again found to be randomly oriented. This process was repeated a number of times in order to provide sufficient data to reduce the effects of random variation. From the data, basic information was obtained on the nucleus and its radioactive decay.

During each run the temperature of the crystal was monitored.

This was done by measuring its magnetic susceptibility, which had previously been determined as a function of temperature by other investigators.[2]

The physical processes which give rise to nuclear alignment may be described as follows. Interaction with the electric fields within the crystal causes the electronic magnetic moment of certain atoms to line up either parallel or antiparallel to a certain crystallographic direction. Then at very low temperatures, where thermal agitation is much less, the coupling between the atomic magnetic moment and the nuclear magnetic moment is strong enough to allow the nuclei also to be pulled into alignment. This coupling cannot begin to overcome the forces due to thermal motion, however, until the temperature falls well below 1° K.

The anisotropic angular distribution of gamma rays from the aligned nuclei can be explained by considering angular momenta. Since the angular momenta of the parent and daughter nuclei as well as that carried away by the gamma ray are fixed by nature, it follows from the principle of conservation of angular momentum and from radiation theory that definite restrictions are placed upon the pattern of gamma-ray emission. The phenomenon is analogous to the radiation from a radio antenna, where anisotropic emission patterns are also observed. This close relationship between gamma radiation and angular momentum makes it possible to utilize gamma-ray intensity data as a basis for deductions concerning the angular momentum of the parent nucleus and the changes occurring during radioactive decay.

Work in this general field is continuing at the Bureau. Attention is now being directed to low-temperature methods of polarizing, rather than aligning, nuclei. Stable nuclei can be employed in experiments of this kind, and a larger number of different nuclei can be studied. The Bureau hopes to obtain additional data of value both to cryogenics and nuclear physics.

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STONE, JOHN: Automatic Control Unit for Model Railroads
KAVANAGH, R. J.: The Application of Matrix Problems to Multi-Variable Control Systems
KRON, GABRIEL: Improved Procedure for Interconnecting Piece-Wise Solutions
GARDNER, GEORGE S., HARRY L. FAIGEN, GREGORY L. GIBSON AND WILBUR S. HALL: Researches on Corrosion and Inhibition. Reaction Velocity in the System Iron: Dilute Acetic Acid at 40° C.
CHANG, S. S. L.: Transient Effects of Supply and Connecting Conduits in Hydraulic Control Systems

PUBLICATIONS RECEIVED

The Editors wish to call attention to these new books which have been received in the JOURNAL office for review. Lack of space prevents publication of full reviews.

MECHANICAL VIBRATIONS, by J. P. Den Hartog. Fourth edition, 436 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $9.00.
A MANUAL OF ENGINEERING GEOMETRY AND GRAPHICS FOR STUDENTS AND DRAFTSMEN, by Hollie W. Shupe and Paul E. Machovina. 347 pages, diagrams, 6¼ × 9½ in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $5.25.
GEOLOGY AND OURSELVES, by F. H. Edmunds. 256 pages, plates, diagrams, 6 × 9 in. New York, Philosophical Library, 1956. Price, $10.00.

Also these republications of classic works, issued in 1956 by Dover Publications, New York:
HYDRODYNAMICS, by Hugh L. Dryden, Francis P. Murnaghan and H. Bateman. 634 pages, 5⅜ × 8 in. 1956. Price, $2.50 (paper).
THE PRINCIPLES OF MECHANICS PRESENTED IN A NEW FORM, by Heinrich Hertz. 274 pages, 5⅜ × 8 in. Price: $1.75 (paper); $3.50 (cloth).
AN ESSAY ON THE FOUNDATIONS OF GEOMETRY, by Bertrand A. W. Russell. 201 pages, 5⅜ × 8 in. Price: $1.50 (paper); $3.25 (cloth).
EXPERIMENT AND THEORY IN PHYSICS, by Max Born. 44 pages, 5⅜ × 8 in. Price, $0.60 (paper).
ASYMPTOTIC EXPANSIONS, by A. Erdélyi. 108 pages, 5⅜ × 8 in. Price, $1.35 (paper).
INVESTIGATIONS ON THE THEORY OF THE BROWNIAN MOVEMENT, by Albert Einstein. 122 pages, 5⅜ × 8 in. Price, $1.25 (paper).

NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, DIRECTOR

THE BETATRON

A recent addition to the Electrical Communications Section of the Museum is a magnetic induction electron accelerator, commonly called a betatron. The device is suggestive of a cyclotron for it speeds its atomic bullets in a spiral path between the poles of a powerful magnet. However, where the cyclotron uses positive particles, the accelerator works on negatively charged electrons.

The advent of this device in 1940 aroused a lively interest among biologists and industrial radiographers. The former perceived in it an instrument for advancing radiation techniques in therapy, while the latter hailed it as opening new prospects of speed and quality in the X-ray inspection of massive steel parts. It is in this latter application that the betatron will appeal to the majority of Museum visitors, for its highly penetrating radiation permits entire assemblies, such as motors, complex castings, and forgings, to be inspected with a speed and precision hitherto unknown.

During the war a number of betatrons were built for radiography. The X-rays generated by the betatron are distinguished for their remarkable penetrating power. The penetration is so high that radiographs can be made to detect flaws in steel from 2 to 12 inches in thickness. The detail is so fine that flaws as small as $\frac{1}{32}$ of an inch deep and .002 inch wide are exposed.

It should be understood that the Museum's betatron can not be demonstrated in operation since the operators must be adequately protected against the radiation, and this would require the construction of a special room with suitable walls.

The theory of how the betatron works is complex but the basic principles are not difficult to comprehend. The principal feature is the X-ray tube, commonly called the "dough-nut," because its shape resembles that of the familiar cookie. It is about 18 inches in diameter, and is made of a ceramic material coated on the inside with palladium. The tube is highly evacuated and permanently sealed.

The dough-nut is placed between the poles of a large electromagnet. Electrons are injected into the magnetic field when the field strength is increasing, and the effect is to cause the electrons to be whirled rapidly around the tube until they attain a velocity just short of the speed of light. In order that the electrons may be shot into their spiral path

along which they are accelerated before they strike the target, it is necessary to use an electron "gun," or injector. This is installed in a sort of horn which projects from the tube. This electron injector contains a filament not unlike that found in the conventional X-ray tube. This filament, when heated, affects the tube current and output.

The electrons are drawn into the tube and sent upon their whirling path by a potential of about 70,000 volts during a period of only about 4 millionths of a second. They are accelerated steadily instead of in jumps across a gap as in the cyclotron. The forces involved are similar to those of the transformer. Just as they reach their maximum energy, after travelling nearly 250 miles almost at the speed of light, the electrons are deflected and caused to strike the target. It is claimed that about 65 per cent of the energy of the electrons is transformed in X-rays. The electrons are then brought out of the tube.

Although the instrument has the appearance of being massive and cumbersome, its flexibility is remarkable. Push-button controlled, it can be adjusted to meet requirements quickly and conveniently.

The dough-nut tube has been generously given by the Machlett Laboratories of Springdale, Conn., and the betatron by the Picker X-ray Corporation, of New York.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, M.D., FOUNDER
WILLIAM G. BATT, D.Sc., DIRECTOR

ACCUMULATION OF HEMATOPORPHYRIN IN ANIMAL TUMORS WITHOUT ENTRANCE INTO THE LIVING CELLS

BY

J. O. ELY AND WILLIAM G. BATT

The sensitivity of paramecia to X-radiation was reported by Figge (1) who with other workers reported that injected porphyrins accumulate in various animal and human tumors (2,3,4). Based on these results it was suggested by Figge *et al.* that porphyrins might be of use in cancer detection and to enhance the effect of X-radiation in cancer therapy.

It was considered of interest to determine if injected hematoporphyrin actually enters tumor cells or is present extracellularly. Before attempting this, experiments were made with two tumors to be certain that these tumors accumulated the hematoporphyrin: the Walker carcinoma No. 256 of the rat, and the Krebs 2 carcinoma of the mouse, the solid form obtained by subcutaneous injection of the ascites cells.

Wistar strain rats of approximately 200 g. bearing Walker No. 256 tumors were injected intraperitoneally with 2–8 mg. of hematoporphyrin and the tumors examined one to 7 days later under illumination in the dark with a Mineralight Long Wave Ultra-Violet Model SL 3660. In no case was the tumor demarcated by fluorescence from the normal tissues. All tissues were deep purple under the illumination.

Mice bearing subcutaneous solid Krebs 2 carcinoma were given 1 mg. hematoporphyrin intraperitoneally. Twenty-four hours later the tumor fluoresced red against a non-fluorescent background of normal tissue.

The deep purple color of the normal rat tissue and of the tumor under illumination suggested that any fluorescence of hematoporphyrin was being masked. The following experiments apparently verified this assumption.

Rats bearing large Walker No. 256 tumors were given intraperitoneally 6 mg. of hematoporphyrin. The following day the hemorrhagic fluid within the tumors was collected and centrifuged. To the sedimented hemorrhagic material glucosol was added. Upon recentrifugation a layer of white blood cells collected at the top of the fluid: the

cells of this fluid fluoresced red due to hematoporphyrin. Like treatment of similar tumors of a rat which had received no hematoporphyrin yielded white cells also but these did not show the red fluorescence. Apparently injected hematoporphyrin is collected in these tumors but the fluorescent color is masked by the color of the animal flesh under inciting illumination.

Having shown that our technical procedure demonstrates the presence of injected hematoporphyrin in the two tumors, the question of its presence within the cell was attacked. For this purpose the Krebs 2 ascites carcinoma of mice was used. Mice with 3 to 6-day-old ascites tumors were given 1 mg. intraperitoneally of hematoporphyrin. From 4 to 48 hours later the ascitic fluid was collected. The fluid always fluoresced red. However, quartz slide preparations of the cell-containing ascitic fluid viewed under a fluorescent microscope failed to show fluorescence of the cells. Instead there was observed a red fluorescent background punctuated by black spherical areas representing the presence of non-fluorescing cells. However, when some of the same ascitic fluid was heated to kill the cells, the cells immediately fluoresced brilliantly.

The addition of a small volume of fluid containing killed cells to a volume of fluid containing living cells yielded a mixture which when observed showed both fluorescent and non-fluorescent cells according to the proportions in the mixture. Moreover, with the proper balance of eosin and hematoporphyrin concentration in such a mixture it was demonstrated by alternate observation under white light and long wave ultraviolet light that the eosin stained cells were the ones that showed hematoporphyrin fluorescence. Thus hematoporphyrin fluorescence, within the limits possible with our equipment, could not be observed in living cells, whereas in the same concentration of hematoporphyrin dead cells fluoresced brilliantly.

It might be considered possible that injected hematoporphyrin is present in the tumor cells in too small a quantity to be observed. For this reason the limit of dilution that would cause fluorescence of dead tumor cells was determined: 1 part in 1 million caused sufficient fluorescence that cells could be observed faintly.

It was noted earlier that the solid subcutaneous form of the Krebs 2 carcinoma of mice accumulated hematoporphyrin and fluoresced red in contrast to the normal tissue, and that in rats having the Walker No. 256 tumor, although no demarcation of the tumor could be observed, hematoporphyrin had accumulated in the tumor. However, teased cell preparations from these tumors did not fluoresce. Glucosol or alcohol extracts of the mouse tumor contained hematoporphyrin.

Investigation was made as to the possibility that hematoporphyrin might enter the cells undergoing mitosis. Krebs 2 ascites carcinoma cells, a small proportion of which had been killed by heat were sus-

pended in glucosol containing a final concentration of 1:1500 hematoporphyrin. Dead cells fluoresced brilliantly. Thirteen mitotic figures were searched out, none of which fluoresced.

Since the tumor cells studied apparently did not contain hematoporphyrin, the question arose as to the reason for the tumor fluorescence. The ascitic fluid of the Krebs tumor contains not only tumor cells but a great number of white blood cells. When the fluid was centrifuged and the supernate discarded, the white blood cells were separated by repeated washing with glucosol and centrifugation: they rose to the top of the fluid. Removal of these cells after repeated washings in an experiment, finally yielded a clear supernate after 6 washings. After 10 washings no more white cells could be separated. The tumor cell mass no longer fluoresced. However, each collection of white cells fluoresced grossly, but individually the cells did not fluoresce. It seems apparent that some hematoporphyrin was adsorbed by the collection of white cells. Dead cells and any debris from dead and disintegrating cells accumulated hematoporphyrin. This may account for the hematoporphyrin in the tumors, or for its accumulation at the site of wounds.

Although hematoporphyrin did not enter the living tumor cells in sufficient quantity to be detected, but was present in the tumors as shown by fluorescence, the possibility of its enhancement of the efficiency of X-radiation of tumors as suggested by Figge was not excluded. To test this possibility two experiments were conducted with the Krebs 2 ascites carcinoma of mice. Since the results of the second experiment verified the results of the first, only one experiment will be noted. One of two groups of mice bearing 3-day implants of 13.6×10^6 Krebs 2 ascites carcinoma cells per mouse was given intraperitoneally 0.1 mg. of hematoporphyrin per mouse. Three hours later the mice of both groups were given 1000 r total body X-radiation. Four days later the average number of tumor cells per mouse was determined. For the X-rayed group the number was 162×10^6 cells, for the hematoporphyrin-treated and X-radiated 160×10^6 cells.

It was observed by Figge that relatively small X-ray doses failed to kill paramecia so long as the paramecia were not exposed to air, but that the organisms died soon after exposure to air. Thus sensitization to low doses of X-rays was oxygen dependent.

In the Krebs 2 ascites tumor the lack of enhancement of X-ray effect by hematoporphyrin may have been due to insufficient oxygen in the peritoneal ascitic fluid. Since light was excluded from the peritoneal cavity of the mouse and the amount of oxygen was limited, perhaps it should have been expected that the hematoporphyrin would not enhance the X-radiation effect.

SUMMARY

That injected hematoporphyrin is accumulated by animal tumors has been verified. However, the hematoporphyrin was not found to enter the living tumor cells.

White blood cells apparently adsorbed hematoporphyrin and any dead cells and cellular debris promptly absorbed it thus suggesting an explanation for its accumulation in tumors.

The effect of X-rays on the Krebs 2 ascites carcinoma of mice was not enhanced by hematoporphyrin.

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NOTES FROM THE BARTOL RESEARCH FOUNDATION

NUCLEAR RESONANCE FLUORESCENCE IN As^{75} *

BY

FRANZ R. METZGER

With gaseous sources of Se^{75} in the form of selenium oxide, resonance scattering from the 265 kev level in As^{75} has been observed.[1]

Preliminary measurements indicate an isotopic distribution of the resonance radiation and thus favor assignment of spin 1/2 to the 265 kev level in As^{75}. Self-absorption experiments indicate a mean life of 10^{-11} second for the 265 kev transition. Although this lifetime is one order of magnitude longer than the prediction of the single particle model,[2] it is considerably shorter than most M1 lifetimes.[3]

* Assisted by the joint program of the ONR and AEC.

[1] For the disintegration scheme of Se^{75}, see, for example, A. W. SCHARDT AND J. P. WELKER, *Phys. Rev.*, Vol. 99, p. 810 (1955).

[2] V. F. WEINHOFF, *Phys. Rev.*, Vol. 83, p. 1073 (1951).

[3] R. L. GRAHAM AND R. E. BELL, *Canad. Journ. Phys.*, Vol. 31, p. 377 (1953).

THORIUM DISPENSER CATHODE *

BY

D. L. GOLDWATER AND W. E. DANFORTH

Encouraging first results have been obtained in the development of a thorium-dispenser type of cathode for magnetron and other power tube use. The cathode is fabricated in three steps: (*a*) a porous tungsten matrix is sintered onto the base material, (*b*) thorium is melted on the surface of the matrix, running in and filling the pores, (*c*) the thorium-impregnated matrix is ground to final size. Average values of d-c. emission current obtained run from 1 amp.cm^{-2}/ at 1520° C. to 2.5 amp.cm^{-2} at 1630. Life tests at these current densities as long as 100 hours have been completed without deterioration of the emitting properties.

The incorporation of thorium in the form of a thorium-molybdenum alloy is also being studied. It is hoped that the loss of thorium from the cathode by evaporation may thereby be decreased. The use of an alloy, of about 70% Th–30% Mo, rather than the pure element, appears to have no undesirable effects on the thermionic properties of this type of cathode.

* This work is supported by the U. S. Navy Bureau of Ships under contract NObsr-72536.

BOOK REVIEWS

THE PURSUIT OF SCIENCE IN REVOLUTIONARY AMERICA, by Brooke Hindle. 410 pages, 6 × 9 in. Chapel Hill, University of North Carolina Press, 1956. Price, $7.50.

The contributions of eighteenth century American scientists, with the exception of those made by Benjamin Franklin, do not occupy much space in the general histories of science. One does not have to seek far for the reason. The inhabitants, almost without exception, were too busily occupied in producing the necessities of life to have leisure for anything else; great endowed institutions were lacking; there were no rich libraries, or ancient universities, or conversation of the learned that graced the Old World. Communications were slow and uncertain. Yet, slowly and almost unobserved, the colonies were drawn into a closer relationship by political ties, and as the obstacles to improvement and advance were removed, an intellectual atmosphere was developed.

Still, it was inevitable that in this new country science should at first be applied to the improvement of material conditions of life, in accordance with the Baconian identification of truth and utility, and the continuing attention to useful knowledge displayed by the Royal Society, which was an important factor in colonial intellectual development. The praise of useful science was sung throughout the colonies, while mathematics and natural philosophy had less appeal. Advances came in technics by trial and error methods rather than through the application of theoretical patterns.

The sciences which showed most progress were botany, since agriculture was the chief support of the people, astronomy, which had a marketable value in its application to navigation, and medicine. Within the fields that attracted the scientifically minded were some important figures, as John Winthrop, John Bartram, Cadwallader Colden, Thomas Jefferson, Peter Kalm, David Rittenhouse, Charles Thomson, in addition to the incomparable Benjamin Franklin.

Dr. Hindle's approach to his subject is described by the publishers as "that of the social historian relating science and scientific developments to the movement of society from 1735 to 1789." He would be a rash reviewer who would complain about his performance, but if a mild, a very mild, suggestion might be offered it is that the author sometimes is so intent upon the individual trees that he loses sight of his forest. For this is a book of distinction. The author has assembled a vast amount of information upon the scientific interests and studies of the generation. He has conducted a thorough investigation of the literature and manuscript material of the period, and brings to light many half-forgotten figures, according to them a new perspective and completeness. Moreover, he maintains a well-balanced judgment throughout. Dr. Hindle's work is worthy of the highest commendation, and we can express the fervent hope that it reaches a wide circle of readers.

T. COULSON

SURFACE AREA, by Lamberto Cesari. 595 pages, 7 × 10 in. Princeton, Princeton University Press, 1956. Price, $8.50 (paper).

The aim of this book is to extend the classical definitions and theorems about curves in space to the subject of surfaces in space. That is, first to define *area* for parametric surfaces in three-dimensional Euclidean space in a way which will coincide with the elementary area for polyhedral surfaces; and second, to obtain relationships between the area of a surface and properties of the functions used in defining the surface. Thus, the early portion of the book reviews the classical material concerning curves and raises the question of how these results can be extended to surfaces. A parametric continuous surface is defined and the definition of area due to Lebesque (1902) is given: namely, the area of a surface S is the lower limit of the elementary areas of polyhedral surfaces P (which are not necessarily in-

scribed in S) approaching the surface S. This definition corresponds to one of the alternate definitions of the length of a curve.

The main portion of the book is then devoted to proving three theorems which relate the Lebesque area of a surface to properties (for example, total variation, absolute continuity, and generalized Jacobian) of the functions used in representing the surface. In addition to these theorems two other definitions of area are considered, and it is shown that the area given by these definitions coincides with the Lebesque area. Also, there is discussion of the problem of an axiomatic definition of area, and a set of axioms for Lebesque area is given.

Although the book is largely self-contained (only basic real variable theory and elementary topology are assumed), the three main results are each obtained only after the preparation of a great deal of formal material. Thus, a detailed reading of the book is intended for specialists in the field; however, the first chapter gives an excellent summary of the results and may be read with profit by anyone interested in the subject.

EMERY THOMAS

AUTOMATION. FRIEND OR FOE?, by R. H. Macmillan. 100 pages, plates, $5 \times 7\frac{1}{4}$ in. New York, Cambridge University Press, 1956. Price, $1.95.

"Once upon a time, a Hindu sage was granted by Heaven the ability to create clay men. When he took earth and water and fashioned little men, they lived and served him. But they grew very quickly, and when they were as large as himself, the sage wrote on their foreheads the word DEAD, and they fell to dust. One day he forgot to write the lethal word on the forehead of a full-grown servant, and when he realized his mistake the servant was too tall! His hand could no longer reach the slave's forehead. . . . This time it was the clay man that killed the sage."

"Is there a warning for us today in this ancient fable?" So queries the author, R. H. Macmillan, an outstanding British engineer who has lectured extensively in this country.

In *Automation. Friend or Foe?* the author has shown how automation is not something which has unsuspectingly been thrust upon us. He shows that more than a century and a half ago we were being subtly subjected to procedures which were to make automation inevitable. In the early days the systems were crude and unpredictable but as time went on the reliability of modern technology was imposed on the devices to bring them to fruition. Today automation is here.

In Chapter I the author speaks of the Development of Automatic Control. Various types of control devices are described. This chapter sets the stage for automation. The next four chapters deal with the Evolution of Automatic Production, Control Systems, Design Problems, the Economics of Automation and Automatic Computers. In these sections the all important attitude of Unions to automation is discussed.

In the last chapter Present Impacts and Future Prospects are discussed.

The impact of automation on our industrial scene is difficult to assess objectively. The view one takes is highly colored by one's station in life. Men have seen automation arrive and wipe out jobs and while it is true others of higher calibre have been created, the immediate effect may be one of anxiety, apprehension, and despair. By the same token owners of large factories have seen automation come in with a tremendous surge in productivity with resultant decrease in costs of processing and labor.

If we could telescope the next 20 years into an instant what wonders would we see as the result of this new force.

One question this reviewer would like to ask of those who, like Mr. Macmillan, have made a serious study of automation is this: With energy sources being continually dried up and the development of new energy sources not keeping pace with the increase in our needs or increase in population, how will automation affect this picture? What can be gained energy-wise with the introduction of automation? Our civilization is geared to a high energy level. Drop the available energy level and civilization must regress. Is it possible that automation may be part of the answer to the problem of decreasing energy stores? The answer would be enlightening.

I. M. LEVITT

AEROELASTICITY, by Raymond L. Bisplinghoff, Holt Ashley and Robert L. Halfman. 860 pages, diagrams, 6 × 9 in. Cambridge, Addison-Wesley Publishing Co., Inc., 1955. Price, $14.50.

This book brings forth the "aeroelastician" —and perhaps many new specialists such as the railelastician in the domain of high speed trains. Actually, the first five chapters are an excellent discussion of elastic continuum in vibration systems. It was Rayleigh who formulated the field of continuous elastic structures to vibration analysis and engineers have been reluctant in extending this valuable theoretical background to design applications. Quite logically the authors introduce at the outset generalized structural analysis in terms of reciprocal influence and elastic coefficients, followed by elastic energy methods, minimum worth of Castigliano's principle as applied to beams and extended structures in airplane wings. From the differential equation of motion of a beam portion, the authors introduce the conception of normal modes associated with finite frequencies, and their orthogonal properties. From this, the beam equation is generalized to normal coordinates. This later is again reviewed from the equation of virtual work through Lagrange's equation, wherein a more formal generalization is given. Obviously the aeroelastician must determine these orthogonal modes for later application with aerodynamic loadings. Computation methods for such are given in Chapter 4. These first four chapters have a real feel in the aerodynamic aspect of the subject as well. For instance, Fig. 1-8, which shows a comparison of wing critical speeds, shows the relative zones for divergence speed, bending-torsion flutter and aileron reversal speed, as functions of sweep forward and sweep back wings.

The second part of the book, starting with Chapter 5, considers the aerodynamic reactions with emphasis on the analysis of small disturbances. As the authors state quite justly, "Moreover, from the aeroelastician's standpoint a defect of nearly all books on fluid dynamics, is their preoccupation with steady flow phenomena." Starting with a short review of hydrodynamic principles, the theory is extended to thin airfoils and finite wings treated first in steady motion and then extending to oscillations in incompressible flow. The work then considers steady supersonic flow, followed by wing forms for these conditions. Chapter 6 continues oscillations in both subsonic and supersonic flow. In this phase much detailed consideration is given to slender straight and swept wings.

The last phase of the work covers an extensive discussion on flutter analysis and dynamic response phenomena, as for systems with time dependent external forces. Further discussion on model interpretation and shape simulation, with methods of testing, is given.

While the book appears at first glance to be written for the specialist in aeroelasticity, actually the book can be considered as a basic and valuable treatise for the entire field of modern airplane design. It is equally valuable to engineers interested in applications of similar techniques in other fields.

RUPEN EKSERGIAN

ELECTROMAGNETIC WAVES, by G. Toraldo di Francia. 320 pages, diagrams, 6 × 9 in. New York, Interscience Publishers, Inc., 1955. Price, $6.00.

This is a translation by the author of an Italian text used in the Physics Department of the University of Florence. It considers principally the study of Maxwell's equations and phenomena relating to the electromagnetic theory of light. In fact, it brings together all of the well-known equations and theorems of Fresnel, Fraunhofer, Green, Poisson, Fermat, Malus-Dupin, Hertz, Huygens, Heaviside, to mention a few.

Reference is made to the outstanding American texts on electromagnetic theory by Stratton, Schelkunoff, and Slater. In some ways Professor di Francia's book is like these, but much abbreviated. In this respect there appears to be excessive economy, for the treatment of each subject is pared down to its most concise mathematical expression. Few words of explanation are provided, and visualization of the quantities involved in the equations is extremely difficult without prior acquaintance.

To prepare the student for the derivations that follow, the author begins with a 40-page introduction to mathematical procedures. At several places elsewhere in the book excel-

lent historical sections are found, and these give an interesting insight into the evolution of the theory. This, the advanced American texts do not generally do.

Illustrating waves in continuous media, several basic developments such as deriving the radiation resistance of a dipole are carried out. But these are few, and usually the principle is given without illustration. In this sense there is a lack of contact with reality. In general there appears to be an excessive emphasis upon the mathematical exercise for its own sake.

One may conclude that the conciseness of the arguments relating to electromagnetic theory in this volume places it in the category of a summary for reference. In this role it might be studied following more rudimentary introductions to the classical theory.

C. W. HARGENS

FUNDAMENTALS OF VIBRATION ANALYSIS, by N. O. Myklestad. 260 pages, diagrams, 6×9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $6.50.

Myklestad's work on vibration analysis is well known and this book is an additional contribution to his previous publications on the subject. In particular, it is an extension of the Holzer method used in a more complicated problem of bending vibrations as occur in wings and continuous beams, propellers and turbine blades.

This book is an interesting contribution on vibration analysis, covering a range usually sufficient for most engineering problems. The text covers certain well known problems in the single degree and in systems of more than one degree of freedom. The author also introduces the more generalized procedure analysis through Lagrange's equations and covers a variety of problems well adapted for this procedure. As in his previous book, the author has a clear-cut discussion of normal modes of vibrations which are illustrated in his analysis of systems of more than one degree of freedom. One other feature of this book is a very good discussion on viscous damping.

The book has little discussion on nonlinear systems and on the operational methods of Laplace's transformations. As the author states, the method is "purely routine when applied to the elementary problems of this book and therefore not suited for developing a physical understanding of the phenomenon of mechanical vibration." The reviewer is entirely in sympathy with this.

The book is an excellent intermediate discussion on vibration analysis and should be well received by all engineers.

RUPEN EKSERGIAN

THE EXPLORATION OF MARS, by Willy Ley and Wernher von Braun. 176 pages, plates, illustrations, 8×11 in. New York, The Viking Press, Inc., 1956. Price, $4.95.

With the closest approach of Mars since 1924 on the horizon public interest in Mars is mounting and will continue to do so until well after the opposition in September. In the meantime to satiate the public's curiosity several books on Mars have appeared.

Of these one of the most powerfully stimulating is *The Exploration of Mars* by Willy Ley and Wernher von Braun. Here are two ranking authorities in their respective fields. Willy Ley is a skillful, articulate writer whose reputation has been firmly established as a popular historian of science. His "Rockets, Missiles and Space Travel" is a classic of his facile pen and he has projected this style into the Mars book. Wernher von Braun is the ranking authority on Space Travel. More than anyone else on earth he is the man responsible for the progress being made in this field and many of the developments going into the Project Vanguard are traceable to this man's genius. To complement these two authorities the most skillful artist working in the astronomical field, Chesley Bonestell, provides the art work.

The book is very roughly divided into two sections. In the first section—the first four chapters—Willy Ley has drawn a very careful picture of the early history of the planet. He speaks of the motions of Mars and correlates them with those of the earth. In Chapter 3 the year 1877 is the highlight. It was in that year that the canals and moons of Mars were discovered. In Chapter 4 all the controversial features of the planet are skillfully covered with strict avoidance of erroneous speculations.

The second section of the Mars book be-

gins with a transition from the planet to space travel and here von Braun takes over with a rare competence to portray the mechanics of a voyage to the planet. It may be recalled that von Braun wrote another book on this precise subject entitled "The Mars Project." In his first book he presented a scientific study investigating the feasibility of a trip to Mars. This was to have been a bold, ambitious adventure involving 70 men and ten rocket ships to be assembled and to leave the space station for Mars. In this brilliant study the "hardware" has been trimmed to more realistic proportions in that only 12 men will travel in two ships to achieve the same goal. If there ever was any doubt as to the technical virtuosity of this man this book erases that completely.

Like the other books in which these writers have participated *The Exploration of Mars* is sound, imaginative, authoritative and presents the details of a trip which this reviewer believes will be consummated by the turn of the century.

If you are a space travel enthusiast, this is one book you cannot afford not to own.

I. M. LEVITT

BOOK NOTES

MICROSCOPIC PETROGRAPHY, by E. Wm. Heinrich. 296 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $6.50.

One of the few texts to stress petrographical techniques, this new book will be a welcome addition to the geologist's library. A knowledge of the techniques of mineralogical optical microscopy is a prerequisite to an understanding of the subject matter.

Containing up-to-date material, this book describes all the major types of rocks in each of the three main groups. Thin sections of the rock are used in the microscope studies, with texturally similar rocks grouped together. The many fine photomicrographs add immeasurably to the value of the book.

The introductory chapter outlines various methods and techniques of microscopic study. The six remaining chapters are divided into three pairs representing aqueous, sedimentary and metamorphic rocks, with one chapter in each pair devoted to general considerations (origin and classification, texture, microstructure, etc.) and the other chapter devoted to detailed descriptions of the types and subtypes of rocks.

Some topics not generally found in texts are discussed in detail, such as: the use of insoluble residues from both carbonate and evaporite rocks; the use of heavy detrital minerals from clastic sedimentary rocks; and the evaporite rocks, with some information on rock borates, rock sulfur, analcite bearing rocks and others.

An excellent index rounds out this unusually detailed text.

CHEMISTRY AND USES OF PESTICIDES, by E. R. deOng. Second edition, 334 pages, 6 × 9 in. New York, Reinhold Publishing Corp., 1956. Price, $8.75.

The change in title from "Insecticides" to "Pesticides" reflects the new approach to the subject, since the first edition of this work was published. Not only has the title been changed—the whole book has been rewritten. The result is an up-to-date treatise on the chemicals used as pest control agents (insects, rodents, weeds, fungi) and as protectants for seeds.

After an introductory chapter giving the history of pest control, the author devotes nine chapters to various types of pesticides (arsenical compounds, copper compounds, sulfur and its compounds, miscellaneous inorganic compounds, plant derivatives, synthetic organic pesticides, and organic herbicides). A final chapter describes the uses of cold, heat, dehydration and radiation as pesticides.

Four appendices strengthen the text: a dictionary of pesticides; a glossary; a listing of tolerances for pesticide residues in or on fresh fruits and vegetables; and a valuable list of official antidotes.

Especially useful as a textbook in vocational agriculture, this volume will also be of value to pesticide manufacturers, food processors and farmers.

A SECOND COURSE OF LIGHT, by A. E. E. McKenzie. 342 pages, diagrams, 5½ × 8½ in. New York, Cambridge University Press, 1956. Price, $3.50.

Intended as a physics textbook for "sixth form" in English schools, this work can be used in U. S. colleges. It is the third in a series of texts on various phases of physics, by the author who teaches at Trinity College, Cambridge.

The subject of light is covered thoroughly in seventeen chapters, each devoted to a single phase of optics. (Examples: spherical mirrors, spectroscopy, color, the velocity of light, wave motion, polarization, etc.) Each chapter is adequately illustrated (there are 296 figures in the book) and is concluded with a list of problems which, if mastered, should assure the student of a good working knowledge of the field. Solutions to the numerical problems are provided in a separate section of the text.

ATOMS AND ENERGY, by H. S. W. Massey. 174 pages, illustrations, 5½ × 8½ in. New York, Philosophical Library, 1956. Price, $4.75.

The author, who worked in America with the British team of scientists on the atomic bomb project, is an authority on the subject of "atoms and energy." In this brief account of the subject, he confines himself to non-technical language, as he relates the developments which preceded the release of atomic energy.

He covers not only the early research on atomic energy, but also the later development of the hydrogen bomb. The material covers the subject through June of 1952.

The book is recommended for the educated layman who wants to comprehend the meaning of atomic energy, but doesn't have the technical background to follow the higher mathematics required for a thorough understanding. Of particular interest is the final chapter on present-day research in pure atomic physics, in which practical applications are not paramount, such as cosmic rays, mesons, the neutrino (the existence of which has just been verified).

OFFICE WORK AND AUTOMATION, by Howard S. Levin. 203 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1956. Price, $4.50.

The business office is due for a revolution, as automation gradually takes over many routine office jobs. To keep up with the latest developments in the field of data processing, the author has written this book for management personnel, so they may evaluate the new processes in terms of their own particular situations.

The book describes the important functions of an office and then sets forth the ways in which automation can perform those functions. The main feature of automation in the office is that its judicious use will enable management to make more accurate analyses of operations and so speed decisions.

The new computers for office work are described and their uses and disadvantages are set forth in a clear, non-technical language that is readily understandable.

The book is recommended to all office managers and other businessmen who—sooner or later—will need to know its contents in order to keep up with their competitors.

BLAKISTON'S NEW GOULD MEDICAL DICTIONARY, edited by Normand L. Hoerr and Arthur Osol. Second edition, 1463 pages, 6½ × 9¾ in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $11.50.

This new edition of an outstanding medical dictionary adds 12,000 new terms and makes 8000 changes over the 1949 edition. At the same time, obsolete terms have been omitted. Some of the fields in which many new terms have arisen are those of chest surgery, psychiatry, cancer research, drugs and nuclear science.

The layout of the book, placing the indices for tables, lists and illustrative plates before the text, is helpful. By placing the section of color and black-and-white plates in the center of the book, the publishers have eliminated the annoyance of having to hold down the pages while examining the illustrations. The explanatory notes on the makeup of the dictionary should be read carefully in order

to get the most benefit from the dictionary itself.

Thumb-indexed for ready reference, and up-to-date, this new edition will be welcomed by those who have come to depend on the previous editions.

FARM ELECTRIFICATION, by Robert H. Brown. 367 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $7.00.

Here is a very practical book on farm electrification. Although it was intended as a textbook for agriculture students, it can well be used by the practicing farmer.

There are two major parts of the book: preparation for the use of electricity; and the use of electricity on the farm. The first covers wiring systems for the entire farm; the second deals with the principles and applications of electricity for light, heat, motors, cooling and water.

Innumerable photographs and drawings enhance the value of the book, as do various tables of electrical ratings, wire sizes, circuit requirements, etc. All conceivable uses for electricity on the farm are covered, from clippers for grooming dairy cows to installations for crop drying and electric cleaners for fruits and vegetables—to say nothing of welders and concrete mixers.

The up-to-date farmer will find much of value in this compact, practical book.

ELECTRONICS IN INDUSTRY, by George M. Chute. Second edition, 431 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $7.50.

In this second edition, recent designs of industrial controls replace older ones and the designs for resistance welding are covered more completely. A chapter on single closed loop systems has been added, giving a basic introduction to servos and automation. In addition, new problems have been added.

ELEMENTARY NUCLEAR THEORY, by Hans A. Bethe and Philip Morrison. Second edition, 274 pages, 6 × 9 in. New York, John Wiley & Sons, Inc., 1956. Price, $6.25.

Nearly one third larger than the first edition, this revised work has been expanded to bring old material up-to-date and to add many new topics. The new subjects include pi-mesons, crystal scattering, low-energy nucleon scattering, high-energy nucleon interactions, polarized nuclear beams, theory of resonance reaction, nuclear cascades and others. The appendix contains an up-to-the-minute list of nuclear species and their masses.

TECHNICAL ASPECTS OF AIR TRANSPORT MANAGEMENT, by R. Dixon Speas. 316 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1955. Price, $8.50.

The title of this book well explains its contents and its purpose. Included in the discussions are factors involved in airline operations, engineering, maintenance and economics. The presentation is such that non-technical people may comprehend it. The book was intended primarily for airline employees interested in knowing something of the technical aspects of their business; however, it serves also as a reference on airline operation and may be used as a text for a course in air transportation.

HIGH FIDELITY CIRCUIT DESIGN, by Norman H. Crowhurst and George Fletcher Cooper. 303 pages, diagrams, 5½ × 8½ in. New York, Gernsback Library, Inc., 1956. Price, $5.95.

The basis of this book lies in a series of articles originally published in *Radio Electronics*. Expanded and combined in a single volume, these papers give the interested reader the benefit of the authors' years of experience designing high fidelity equipment. The treatment is informal, but practical, so that the hi-fi fan can either build his own set, test the components of the one he has, or add to his existing equipment after a careful reading of the various chapters.

WEATHER ANALYSIS AND FORECASTING. VOLUME I. MOTION AND MOTION SYSTEMS, by Sverre Petterssen. Second edition, 428 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $8.50.

The revised edition of this work appears in two volumes instead of one. Volume I, now available, emphasizes the dynamics of atmospheric processes—cyclones, anti-cy-

clones, fronts and upper waves. Volume II will emphasize the thermodynamics of atmospheric processes.

Volume I, although it devotes considerable space to forecasting procedures, may be used as a text for courses not specifically directed at that branch of meteorology.

The revision includes the new techniques and new knowledge acquired by researchers since 1940, when the first edition appeared.

TABLES OF WEBER PARABOLIC CYLINDER FUNCTIONS, edited by J. C. P. Miller. 233 pages, $8\frac{1}{2} \times 10\frac{3}{4}$ in. London, Her Majesty's Stationery Office, 1955. Price, $11.68 postpaid. (Available in U. S. from British Information Services.)

The tables in this useful volume were computed by Scientific Computing Service, Ltd., and their publication was sponsored by the National Physical Laboratory. The mathematical introduction to the tables (some 92 pages of text) was written by Dr. Miller, who also supervised the computations. The equation whose solutions are covered by these tables is

$$\frac{d^2y}{dx^2} + (\tfrac{1}{4}x^2 - a)y = 0.$$

In the introductory material Dr. Miller gives the mathematical development of the theory of parabolic cylinder functions, and includes an extensive collection of formulae. Then follows a description of the method used in preparing the tables, which occupy slightly more than half the book.

The tables themselves (six in all) will be of great value in the solution of a wide variety of problems in mathematics, physics and engineering.

MODERN CHEMICAL PROCESSES, VOLUME 4, by the Editors of *Industrial & Engineering Chemistry*. 202 pages, illustrations, 8×11 in. New York, Reinhold Publishing Corp., 1956. Price, $5.00.

Volume 4 in this useful series contains 19 papers, each devoted to a recently developed chemical process now on full-scale production. Each article contains background material, a description of the actual plant installation complete with design details of non-standard items, a description of plant operation, economics, personnel and future prospects.

Together with the previous three volumes, this volume (and those to come at two-year intervals) will be a valuable reference for the latest developments in the rapidly expanding chemical and process industries.

CHEMICAL PROCESS INDUSTRIES, by R. Norris Shreve. Second edition, 1004 pages, diagrams, 6×9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $11.50.

This second edition of a widely used text and reference book brings the work up to date of January 1954. The hundreds of tables and flow charts, which give a clear picture of each process, have been carefully checked by the author and by experts in the respective industries. One change in this edition removes the problems from the ends of chapters and places them in a special appendix. Those who used the first edition (1945) as a "bible" will need to replace it with this modern edition.

CHEMICAL MARKET RESEARCH IN PRACTICE, edited by Richard E. Chaddock. 196 pages, $5 \times 7\frac{1}{4}$ in. New York, Reinhold Publishing Corp., 1956. Price, $3.00.

Based on a series of lectures presented both at Case Institute of Technology and at the University of Delaware, this book offers sixteen papers by 22 experts on various phases of chemical market research. The aim of the book is to provide material for a college course, but it can also serve to give executives a better insight into the practical values of market research in the chemical industry.

CURRENT TOPICS

Detection of the Neutrino.—The Los Alamos Scientific Laboratory, operated for the Atomic Energy Commission by the University of California, has announced that a group of scientists led by Frederick Reines and Clyde Cowan, Jr., has collected experimental evidence for the existence of the free neutrino, a particle without charge and with vanishingly small mass. The neutrino's existence was confirmed at the Savannah River Plant.

Over 20 years ago, Nobel Laureates Enrico Fermi and Wolfgang Pauli theorized on the existence of such a particle in order to account for the mysterious disappearance of energy from a radioactive process known as beta decay. However, the neutrino has hitherto escaped the direct detection deemed necessary to prove its existence in the free state away from the radioactive atom from which it is emitted.

Cowan and Reines believe they first observed the neutrino in the spring of 1953 when, with the help of another group of scientists, they set up a large and rather novel liquid scintillation system as a sensitive detector for the tiny particle near an atomic production reactor at the Hanford, Washington Plant of the Atomic Energy Commission.

Although the evidence obtained at Hanford was indicative of the neutrino's existence as a particle in the free state, the experiment was not entirely conclusive. The physicists returned to Los Alamos and devised a more complex detecting system which would, they hoped, better select the neutrino signals from the confusing background due to radioactivity and cosmic rays.

Last fall the new equipment was taken to the Savannah River Plant, South Carolina, operated by the Du Pont Company for the Atomic Energy Commission, and set up deep underground near one of the large reactors there. After several months of working with the equipment, the scientists feel they have now checked each important characteristic of the neutrinos caught in their detector.

The neutrino interacts only very weakly with material, and could be expected to penetrate a distance of many light years through solid matter. Consequently, the detector developed by the Los Alamos scientists is of extraordinary design. In it more than 100 gallons of ordinary water containing a dissolved cadmium salt served as a target for the neutrinos coming from the reactor. This target was "watched" by a scintillation system which is in principle like the scintillation detector used by uranium prospectors but which contained over 1,000 gallons of sensitive liquid and 330 large photo-electric eyes, called photomultiplier tubes.

Despite its huge size and the many billions of neutrinos from the reactor, which passed through it each second, only a few neutrino captures were observed in the target water each hour.

The discovery marks the first time that man has knowingly caused a direct reversal of the radioactive process called beta-decay. In the present experiment, normally stable protons (hydrogen atoms in the water) were made to absorb a neutrino, emit a positive electron, and become neutrons. Thus, a particle with the expected properties of the neutrino has been detected in an inverse beta-decay reaction, the theory of Fermi and Pauli may be con-

sidered as proved, and nuclear scientists may accept with some confidence the further theories which have been developed involving the neutrino. They may also avail themselves of the neutrino to assist them in the understanding of the atomic nucleus and of the constitution of the universe.

Transistors Used to Detect Radiation.—The tiny transistor, an electronic miracle that may revolutionize radio and television equipment design, today is being applied to an important phase of atomic energy development. General Electric engineers at the Hanford atomic plant are using transistors in prototype radiation detection instruments which may replace detection devices now in use throughout much of the atomic industry.

Utilizing transistors instead of larger vacuum tubes wherever possible, members of the General Electric Radiological Sciences Instrument Development Unit at Hanford have come up with transistorized instruments that are lighter, smaller, more rugged, and less sensitive to temperature and humidity changes than many detection devices now available. Tests also have proved the new instruments to have a battery life many times greater than vacuum types.

Transistors are small electronic devices which perform functions similar to those achieved with electron tubes. Their minute size lends them to miniaturization of electronic equipment, thus making possible savings in space and materials.

Prototypes being built at Hanford feature transistorized alpha meters, beta instruments, gamma detection instruments, and devices capable of detecting all three types of radiation.

Equipped with the new detectors, radiation monitors will be able to move freely from one room to another and from outdoors to indoors without humidity or temperature changes affecting the operating efficiency of these instruments. In tight quarters, the compact transistorized detection instruments will permit measurements to be made more easily than with bulky vacuum detectors.

Generally speaking, transistorized detection instruments weigh far less and are much smaller than their vacuum tube forerunners. They have been designed for low maintenance cost and for ease of calibration.

Vinyl Fabric Darkening Drapery.—A new flame-resistant, vinyl-coated fabric for draperies to darken window areas in schoolrooms and auditoriums, primarily for audio-visual work, has been developed by the Du Pont Company's Fabrics Division. The opaque drapery is color-patterned and textured on the inner side. Neutral beige faces the window. Opacity is supplied by an intermediate coating of black vinyl. Du Pont "Tontine" vinyl drapery material is of a weight and softness designed to drape gracefully over large expanses of glass and to draw open into minimum space. Like regular drapery fabrics, it can be sewn and pleated, and can be hung on the same types of hardware.

"Tontine" drapery has all the ruggedness for which vinyl fabrics are noted. In addition, dust shakes off easily since the surface is non-porous and the finish unusually dry.

The Du Pont laboratory at Newburgh, New York, developed this darkening drapery material to meet the following specifications: permanent flame resistance, high resistance to deterioration in tearing and tensile strength, discoloration due to fading, darkening or chalking, tackiness under any condition, stiffening and embrittlement, shrinking or stretching.

Du Pont "Tontine" darkening drapery is 54" wide plus selvage. It comes in gray, green, beige, and brown in four patterns—Lorient, gently waving vertical lines; Barony, a tweed effect; Linen Crash, a bold linen treatment; and Shetland, a block pattern.

High Temperature Resistant Paint.—A new and unique high temperature resistant paint has been developed at the Corps of Engineers' Research and Development Laboratories, Fort Belvoir, Va. Designed primarily for field application for the protection of diesel engine exhaust systems, it has proved to be even more satisfactory for plant application.

Senior Project Engineer, Emil J. York, states that the paint can be applied by either brush or spray and will air dry to handle within one hour. In its air-dried state, it provides excellent protection from rust and corrosion and after complete curing the protection afforded is unlimited. The paint can be applied and cured in place by merely using the engine or it can be applied and cured immediately as in plant operation.

In tests conducted by a large manufacturer of heavy construction equipment the paint has remained in excellent condition after more than 18 months' exposure at their proving grounds. Tests at the Engineer Laboratories show the paint to be in perfect condition after 1200 hr. exposure to temperatures in excess of 1400° F.

Major requirements for satisfactory performance are a clean and oil- or grease-free surface, preferably blasted, and a controlled maximum dry film thickness of 2.5 mils.

A military specification is being prepared on this material and its use will be mandatory on Corps of Engineers equipment.

Radioactive Tracer Experiment of Sewage Field.—Radioactive isotopes have been successfully used for the first time to trace the dispersion of sewage effluent in ocean waters. Hyperion Engineers is now designing more than 40 million dollars' worth of additions and improvements for the Los Angeles sewage system and Hyperion Treatment plant. The unusual application is part of oceanographic studies designed to insure against beach pollution when the modified plant, with its increased flow of sewage, is placed into operation.

The experiment had the approval of U. S. Public Health and Atomic Energy Commission officials as well as State and local health agencies. It is noted that radioactivity in the effluent, even at the point of introduction into bay waters, was substantially less than the concentration permitted for drinking water as established by the National Bureau of Standards.

The actual tracer experiment was conducted jointly by Hyperion Engineers of Los Angeles, Calif.; Nuclear Science and Engineering Corporation of Pittsburgh, Pa.; and the Allan Hancock Foundation of the University of Southern California. Introduction of the radioactive material—scandium-46—was performed by NSEC technical personnel. It was mixed with effluent from the Hyperion Plant and discharged into the sea through the existing outfall.

Scientists aboard the laboratory ship, *Velero IV*, took radioactive measurements at various depths and positions to determine dilution and rate and direction of diffusion. This phase covered about 36 hr. and an area of 25 sq. mi.

The equipment necessary for introducing the radioactive tracer included a shielded tank with agitator, flow meter, valves and controls, so that the

proportions of scandium-46 and sewage effluent would remain constant throughout the one-hour period of introduction.

Selection of scandium-46 was made after a thorough search for a suitable tracer. Scandium-46 was deemed satisfactory since it meets all requirements for radiological health standards, has a short half-life and its cost is not prohibitive. The twenty curies required for the experiment were obtained from the Oak Ridge National Laboratory.

Oil Pressure Measured by Remote Control.—American jet planes in the arctic will get off the ground faster and fly with greater safety as the result of a revolutionary new oil pressure measuring system that eliminates bulky and dangerous oil lines.

Perfected by engineers of Thomas A. Edison, Incorporated, in cooperation with specialists at Wright Field, the unique system, which consists only of a pressure transmitter and indicator, substitutes electric wires for conventional oil lines and gives an immediate and accurate reading of oil pressure under extremes of temperature.

According to Henry G. Riter, 3rd, president of Edison, the new system, developed after more than six years of research, eliminates the danger of oil congealing in the lines in cold weather, and also does away with the danger of an engine's losing its lubricant once a line has been ruptured in battle.

The Defense Department has already specified the Edison system for its newest jet airplanes.

The company is continuing its research to find other military and civilian applications for the new system, according to Mr. Riter.

Edison's new transmitter will indicate pressures in temperatures as high as 450° F. and as low as − 65° F. Thus instantaneous and accurate readings are assured under weather conditions in any area of the world and at any altitude.

Edison's pressure transmitter can be mounted directly on aircraft engines because its construction permits operation even under conditions of severe vibration. Direct installation has been impossible with other types of pressure transmitters.

The pressure transmitter operates with no contact between moving parts and has no area of friction. A metal bar, attached to a diaphragm, moves in and out of a magnetic field as the diaphragm responds to changes in oil pressure. This is translated into an electrically transmitted signal which activates the indicator on the plane's instrument panel.

Radio Signals from Venus.—On June 4, 1956, scientists at the Naval Research Laboratory announced the detection of weak radio transmissions emanating from Venus, the Earth's sister planet. The very weak radio emissions from Venus had been observed for several weeks before that date by three NRL radio astronomers, Cornell H. Mayer, Russell M. Sloanaker, and Timothy P. McCullough, using the 50-foot radio telescope of the Naval Research Laboratory and specially designed electronic equipment at the high radio frequencies. The Venus transmissions were received on a wave band of approximately 10,000 megacycles.

On June 22 Venus is at its closest point to the Earth in the planet's orbit. It will be another year and a half until Venus will be so close again. This "close" distance means that on this publication date Venus is a mere 27 million miles away from Earth. However even at 27 million miles Venus is then closer to the Earth than any other major astronomical body

execept the moon. This is a good time to study our closest neighbor planet, and Naval Research Scientists are doing just that. Data are presently being collected and evaluated and if significant will be released.

The surface of Venus cannot be seen through telescopes because the planet is covered by a thick blanket of white clouds. It is sunlight reflected from these clouds that makes Venus such a brilliant object in the sky.

Previous information collected on Venus tends to show that life as we know it here could not exist. According to Mr. Cornell H. Mayer of NRL, Venus has a surface temperature of more than 212° F., or hotter than boiling water! Moreover, spectroscopic observations show that there is neither water vapor nor oxygen in the Venus cloud formations. What elements compose the Venusian clouds is a matter of conjecture.

Arctic Shelter Developed.—An Arctic shelter which has been erected in 90 minutes by seven men under ideal conditions has been developed by the Corps of Engineers' Research and Development Laboratories, Fort Belvoir, Va.

Capable of withstanding sustained winds of 100 mph. and snow loads up to 75 psi., the building has been adopted by the Army for use as barracks, communications shelters and first aid stations after highly successful tests at Fort Churchill, Canada, and Big Delta, Alaska.

Three-inch thick stressed-covered panels with fir plywood surfaces glued to the wood edge banding and interior framing are utilized. Cavities, formed between these members, are lined on the inner face by aluminum foil on kraft paper. The foil extends up the sides of the framing to form a cup, which is filled with 2½-lb. density fiber glass insulation. The roof construction consists of skin-stressed fir plywood beams supported on the bearing walls.

Fir plywood faces are attached to the framing by phenol-resorcinol glue and nails at three-inch intervals. A single fastener—a wedge clip—connects all panels together. Tongue and groove joints are used throughout the panel envelope. Recessed in the groove, the hardwood tongue makes a weather-tight joint by compressing a gasket at the bottom of the groove.

Wall panels measuring 4 × 8 ft. and weighing 96 lb. have tongues on three edges and a groove with a recess gasket on one of the long edges. Three interchangeable types—plain, window, and door—are provided. The building can be varied as to the number of windows and doors by increasing or eliminating these panels.

Triple-glazed, hermetically sealed plastic windows are used in the building. They can be removed and replaced by other types or by screening in the summer. Ventilation is provided by slots cut in the framing under each window. A rotatable cover controls the amount of ventilation desired. Screen doors are also provided for summer use.

The floor panels are four by ten feet and weigh 129 lb. Framing members run longitudinally inside the panel and consist of ¾-in. × 2½-in. members at 12-in. centers. In an assembled building, the floor is covered by a tempered Masonite wearing surface. Special roof panels are provided for the corners, while all other panels are identical and weigh 115 lb. each.

A vestibule is provided by 2-in. thick insulated partition panels with a door on one panel. This partition can be placed to satisfy the particular requirement for vestibule and cold room storage or utility.

Furnished with the building is a 70,000 BTU oil burning space heater which adequately maintains a 70-deg. temperature in ambient temperatures as low as 65 deg. below zero.

At Fort Churchill the 20 × 48-ft. structure, which is air-droppable in a disassembled condition, was erected in 65 man-hours by unskilled troops working in temperatures 35 deg. below zero. A modified design of the building sheltered personnel participating in the Navy's recent Antarctic expedition.

Radioisotopes from Liquid Waste.—General Electric Company scientists have found how to remove a valuable radioisotope from liquid waste at the Hanford plutonium plant. The technique eventually may save taxpayers millions of dollars, they believe.

By removing radioactive cesium (Cs 137) from this waste material, they retain in concentrated form an isotope potentially valuable for medical and industrial uses and render the remaining waste much less expensive to dispose of.

Dr. A. B. Greninger, G-E scientist and manager of the engineering department at the Hanford atomic plant, said that because of this removal process, waste that used to be stored in expensive steel and concrete tanks can be pumped into pits scooped out of Hanford's sandy soil. Only the cesium-enriched residue then has to be stored in the costly tanks.

Dr. Greninger said that the radiocesium can't be discharged into the ground because it would take 33 years to lose half of its rather intense radioactivity. Thus, it might survive throughout a long and devious underground trip to the Columbia River and damage aquatic organisms there.

But he pointed out that its long half-life makes it potentially valuable in medical therapy, inspection of opaque materials and food sterilization, since it gives off X-ray type radiation. The long half-life makes it advantageous because it minimizes the need for recalibration and replacement.

G-E operates Hanford for the Atomic Energy Commission.

Miniature Backward Wave Oscillator.—The first miniature backward wave oscillator has been developed by Varian Associates, Palo Alto, Calif. The Varian Model VA-161 combines low power requirements, small, compact size and light weight with rugged construction to provide the only miniature backward wave oscillator now available. Designed for modern, miniaturized equipment the new Varian tube is instantaneously tuned by changing voltage.

Used for radar systems, signal generators, search receivers and related microwave equipment, the VA-161 tube operates over the normal 8.5 to 9.6 kMc radar band on less than 300 volts, making possible the use of existing radar system power supplies.

Varian's new backward wave oscillator contains a permanent magnet which weighs less than 5 lb., eliminating the need for an electromagnet and its associated power supply. The overall size of the tube is approximately 4 in. long with a ¾-in. diameter. Because of its metal and ceramic construction, this backward wave oscillator will withstand severe shock and vibration, according to the manufacturer. The power output of the Model VA-161 is said to be smooth across the entire tuning range, with relatively minor fluctuations.

First Private Nuclear Reactor Operating.—Operation of the first nuclear reactor built for private industrial research began in June. The 50,000-watt research reactor, designed and

built by Atomics International, a division of North American Aviation, Inc., for the Armour Research Foundation, is intended solely for peacetime use by industry.

Located on the campus of the Illinois Institute of Technology, the reactor opens an entirely new field in industrial research and development by providing an on-the-spot source of high-energy gamma rays and neutrons. Short-lived radioisotopes, useful in medical, industrial and scientific research, will be available locally from the reactor. It will not be used to generate electrical power.

The "solution type" reactor uses as "fuel" enriched uranium dissolved in about four gallons of water. Atomic fission, the "splitting" of atoms which produces radioactivity and neutrons, takes place in the reactor core, a steel sphere about one foot in diameter.

The reactor core is surrounded by an $8 \times 5 \times 5$-ft. stack of graphite bars which acts as a reflector. Dense concrete, 5 ft. thick, forms the exterior shield.

Rate of fission in the reactor is controlled by four boron rods. When placed near the reactor core the rods absorb neutrons, thus stopping the fission process. When the rods are withdrawn fission begins again.

A particular feature of the reactor is its "self contained" design which confines all the radioactivity within the shield. No gaseous or liquid materials are exhausted or discharged.

The reactor will be used in many fields including biology, petroleum technology, metallurgy, electronics, textiles and chemistry. Of particular interest are studies in the atomic radiation of foods to extend the length of time they can be kept edible. Among the processes offering promise are the pasteurization of fresh meat to extend its shelf life, "cold" sterilization of meats and vegetables for indefinite storage, deinfestation of grains and flour and prevention of sprouting in potatoes and onions.

Radioisotopes provide some of the most useful and promising applications of the reactor. Such "short-lived" isotopes have not been widely used because of the time delay and resultant radioactivity loss if moved over long distances.

Because radioisotopes send out "signals" that can be detected and measured, a small quantity of material can be radioactively "tagged" and its invisible course traced in metals, plants and animals. Flaws in the casting and welding of metals can be easily detected in this manner. Used as tracers that can be picked up by detecting instruments, radioisotopes are revealing the secrets of how plants grow and the most efficient ways they can be fertilized and irrigated.

In addition to the reactor for the Armour Research Foundation, Atomics International has contracted to build a research reactor for the Atomic Energy Research Institute of Japan and a medical reactor for the University of California at Los Angeles Medical Center.

In the nuclear power field, Atomics International has designed and is building a sodium graphite power reactor which will go into operation in California the latter part of this year. This is the SRE, or Sodium Reactor Experiment, a nuclear power reactor being built for the Atomic Energy Commission as part of the Commission's program to develop economical power from atomic energy.

Two solution-type reactors, built by Atomics International for the AEC for nuclear research work, have been in operation in California for several years. One is operating at Atomics International's facility in Los Angeles

and the other at the Livermore Research Laboratory.

Run-Off Table Rolls with Ductile Iron.—Run-off table rolls of ductile iron, installed at Inland Steel's plant in East Chicago, Ind., more than three years ago, have given six times the service life of cast iron rolls to date. They are still in perfect condition, promising an even longer life.

In a recent comparison report, the Inland Steel company announced that cast iron rolls had to be replaced every six months. The conveyor rolls, which carry red hot steel strip to cooling beds, are intensely hot one minute and then suddenly cooled by cold water. When cast iron was used, cracking generally occurred within a half-year. Inland attributes ductile iron's performance record to its superior resistance to thermal shock, and to greater elevated temperature strength and over-all toughness.

The change from cast iron to ductile iron rolls has reduced down time and production loss, while saving on maintenance and material costs, Inland Steel reports.

Inland's ductile iron rolls are produced by the Youngstown Foundry & Machine Co., Youngstown, Ohio.

Ductile iron was discovered and developed by the International Nickel Company. Ductile iron bridges the gap between cast iron and steel, combining the low cost, machinability and process advantages of cast iron with the strength of steel. Production has been licensed by International Nickel to about 350 companies throughout the world.

Pocket Microscope-Telescope.—Plant operating engineers, lab technicians, design and research personnel will find dozens of practical uses for a handy pocket instrument, combining a 50 power Microscope and a 10 power Telescope, placed on the market by Edmund Scientific Corp., Barrington, N. J. Approximating a fountain pen in size, when used as a microscope it will enlarge 50 times. It is useful for close-up examinations of small parts and components of plastic, wood or metal. It is also handy for spotting, checking, inspecting on machining operations, threads, chamfers, wear on cutting tools, for studying insects, crystals, food particles, etc. It is focussed by tilting, and a metal reflector throws light on the subject.

When used as a Telescope, the metal reflector is unscrewed, and the eyepiece tube and sight are drawn out. Distant objects are brought ten times closer. It is used for chimney, roof, wire inspections, for distant gauge readings, etc. The instrument is equipped with a clip to hold it in a coat or shirt pocket. Finished in chrome and black, it will last for years.

Expansible Van.—A truck-mounted van which can be expanded to almost twice its seven-foot travelling width has been developed by the Corps of Engineers' Research and Development Laboratories, Fort Belvoir, Va., to provide the Army with mobile units for photomapping operations.

Mounted on a standard 2½-ton truck chassis, the van can be expanded to 13½ ft. by two men in less than five minutes to provide a working area of 230 sq. ft. If the unit must be moved to a more strategic site, the sides can be retracted in the same amount of time.

Each side wall, built on five outrigger beams which engage sprockets mounted on a longitudinal shaft, is extended by a cranking motion. When the sides are fully extended, hinged floor and roof sections are swung into position. After the end panels are

closed, the sides are retracted slightly to give weather-tight seals to all joints.

The weight of the van has been kept to a minimum by the use of specially fabricated steel sections, aluminum panels and tread plate flooring, and aluminum shapes and fittings whereever practical. The walls and ceiling are lined with plywood.

A plastic coating is applied to the roof exterior to reflect sun rays and reduce heat absorption. The entire body is insulated with fiberglas and is fitted with double-glazed, high altitude, non-frosting windows equipped with blackout panels.

Fluorescent lighting is provided to obtain maximum illumination and a minimum of radiant heat. The lighting and power receptable system is equipped with an automatic blackout device to break lighting and receptable circuits when the doors are opened. All wiring is for 120/208 volt, 3-phase, 4-wire power from an outside source. Generally this power is provided by a trailer-mounted generator which is towed by the truck.

A photomapping company of an Army Topographic Battalion will be furnished with a "train" of eleven expansible vans. Each will be heated and air conditioned for operation under a wide variety of climatic conditions.

The train will provide facilities for computing, plotting, drafting, editing, revising, rectifying and printing aerial photographs; mosaic and multiplex compilation and supply operations.

During World War II, photomapping companies were forced to transport their precision instruments in cargo trucks and work in tents or battered buildings. This resulted in an excessive loss of time from technical operations.

The Boyertown Auto Body Works of Boyertown, Pa., assisted in the development of the van and built pilot models for early tests. The van has also been manufactured by the McCabe-Powers Auto Body Company of St. Louis, Mo.

Because of its unique construction features, the expansible van may be suitable for use as mobile maintenance, barber, optical, and map reproduction shops; dental and X-ray clinics; operating room; and radio, radar and fire control centers.

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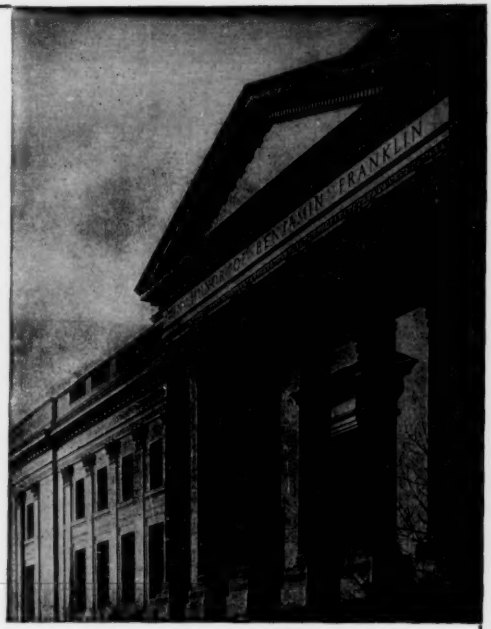

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